Nonperturbative vacuum effect in the quantum field theory of meson mixing

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Replacing the perturbative vacuum by the nonperturbative vacuum, we extend a recent development of a quantum field theoretic framework for scalar and pseudoscalar meson mixing. The unitary inequivalence of the Fock space of base (unmixed) eigenstates and the physical mixed eigenstates is investigated and the flavor vacuum state structure is explicitly found. This is exploited to develop formulas for two flavor boson oscillations in systems of arbitrary boson occupation number. We apply these formulas to analyze the mixing of η with η' and comment on the other meson-mixing systems. In addition, we consider the mixing of boson coherent states, which may have future applications in the construction of meson lasers.

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I. INTRODUCTION

The study of mixing transformations plays an important part in particle physics phenomenology [1]. The standard model incorporates the mixing of fermion fields through the Kobayashi-Maskawa (CKM) [2] mixing of 3 quark flavors, a generalization of the original Cabibbo [3] mixing matrix between the d and s quarks. In addition, neutrino mixing and oscillations are the likely resolution of the famous solar neutrino puzzle [4]. In the boson sector, the mixing of K^0 with K^0 via weak currents provided the first evidence of CP violation [5] and the $B^0\overline{B^0}$ mixing plays an important role in determining the precise profile of a CKM [2,3] unitary triangle [6] in Wolfenstein parameter space [7]. The η η' mixing in the $SU(3)$ flavor group also provides a unique opportunity for testing QCD and the constituent quark model. Furthermore, the particle mixing relations for both the fermion and boson case are believed to be related to the condensate structure of the vacuum. The non-trivial nature of the vacuum is expected to hold the answer to many of the most salient questions regarding confinement and the symmetry breaking mechanism.

The importance of the fermion mixing transformations has recently prompted a fundamental examination of them from a quantum field theoretic perspective [8]. A similar analysis in the bosonic sector has also been undertaken [9]. However, more recent analysis [10] of the fermion mixing indicated that the previous result [8] based on the perturbative vacuum is only an approximation with respect to the exact one based on the nonperturbative (flavor) vacuum. In this work, we show that the same is true for the bosonic sector. Upon the completion of our work, we notice that the same conclusion was also drawn in recent literature [11]. In our work, however, the orthogonality between mass and flavor vacua is shown in a straightforward algebraic method rather than solving a differential equation for the inner product of two vacua as presented in [11]. As evidenced in the previous literature [8,10–12], the method of using a differential equation to prove the unitary inequivalence between the two Fock spaces has been known for some time and our algebraic method is a new development in this respect. Moreover, we analyze the structure of the nonperturbative flavor vacuum in a great detail contrasting to the fermion case. The details of flavor vacuum, its perturbative expansion in the mixing angle, and also some clarifying remarks on the Green function method and the arbitrary mass parametrization are summarized in the accompanying Appendices.

We begin in Sec. II with investigation of the vacuum structure using the relation between the base eigenstate and the physical mixed-eigenstate fields. We derive the representation for the Pontecorvo mixing transformation for the boson case and explicitly calculate the flavor vacuum state structure in the quantum field theory (QFT). We then investigate the unitary inequivalence of the two Fock spaces — one is the space of mass eigenstates and the other is the space of flavor eigenstates. In Sec. III, the ladder operators are constructed in the mixed basis. These are used to derive time dependent oscillation formulas for 1-boson states, n-boson states, and boson coherent states. Consequences from the replacement of the perturbative vacuum by the exact nonperturbative (flavor) vacuum are demonstrated. Section IV is devoted to study specific cases in our formalism, such as the $\eta\eta'$ system. We show the numerical differences between the two results: one from the perturbative vacuum and the other from the nonperturbative vacuum. Conclusions and discussions follow in Sec. V. In Appendix A, we present a derivation of an explicit expression for the flavor vacuum operating the ladder operators of particle and antiparticle to the vacuum of mass eigenstates. In Appendix B, we discuss the region of validity for a perturbative expansion of the flavor vacuum. In Appendix C, we make some clarifying remarks on the Green function method and the arbitrary mass parametrization discussed in recent literature [10,11,13].

II. THE MIXING RELATION AND VACUUM STRUCTURE

We start our analysis by considering the Pontecorvo mixing relationship [14] for two fields:

$$\phi_\alpha = \cos\theta\varphi_1 + \sin\theta\varphi_2,$$

$$\phi_\beta = -\sin\theta\varphi_1 + \cos\theta\varphi_2, \tag{2.1}$$

where $\varphi_{1,2}$ are the free fields with definite masses $m_{1,2}$ and $\phi_{\alpha,\beta}$ are the interacting fields with definite flavors α, β,

respectively. The above mentioned relationship naturally arises by considering the mixing problem for the two quantum fields with the Lagrangian of the form

$$L = L_{0,\alpha} + L_{0,\beta} - \lambda(\phi_\alpha^\dagger \phi_\beta + \phi_\beta^\dagger \phi_\alpha), \tag{2.2}$$

where $L_{0,\alpha(\beta)}$ are the free flavor-field Lagrangians [i.e., $L_{0,\alpha(\beta)} = \frac{1}{2}(\partial \phi_{\alpha(\beta)}^\dagger \partial \phi_{\alpha(\beta)} - m_{\alpha(\beta)}^2 \phi_{\alpha(\beta)}^\dagger \phi_{\alpha(\beta)})$] and λ is the coupling constant responsible for mixing. It is straightforward to show that the above Lagrangian can be immediately diagonalized by the transformation given by Eq. (2.1) with an appropriate choice of mixing angle θ [10]. The parameters of the diagonalized Lagrangian can then be expressed in terms of flavor-field masses (m_α, m_β) and interaction constant (λ); i.e.,

$$\tan(2\theta) = \frac{-4\lambda}{m_\alpha^2 - m_\beta^2} \tag{2.3}$$

and

$$m_{1,2}^2 = \frac{(m_\alpha^2 + m_\beta^2) \pm \sqrt{(m_\alpha^2 - m_\beta^2)^2 + 16\lambda^2}}{2}. \tag{2.4}$$

The free mass fields $\varphi_{1,2}$ can be written explicitly as usual:

$$\varphi_i = \sum_{\vec{k}} \frac{1}{\sqrt{2\epsilon_i(k)}} (u_{\vec{k}i} a_{\vec{k}i} e^{-ikx} + v_{\vec{k}i} b_{\vec{k}i}^\dagger e^{ikx}), \tag{2.5}$$

where $a_{\vec{k}i}$ and $b_{\vec{k}i}$ are respectively the particle and antiparticle ladder operators for the free mass fields and they satisfy the standard equal-time commutation relationships:

$$[a_{\vec{k}i}, a_{\vec{k}'i'}^\dagger] = \delta_{\vec{k},\vec{k}'} \delta_{i,i'},$$

$$[b_{\vec{k}i}, b_{\vec{k}'i'}^\dagger] = \delta_{\vec{k},\vec{k}'} \delta_{i,i'}. \tag{2.6}$$

Here, $kx = k_0 x_0 - \vec{k} \cdot \vec{x}$ and $\epsilon_i(k) = k_0(k) = \sqrt{\vec{k}^2 + m_i^2}$. For the spin-0 case, free mass-field amplitudes $u_{\vec{k}i}$ and $v_{\vec{k}i}$ are just numbers, i.e., $u_{\vec{k}i} = v_{\vec{k}i} = 1$. The interacting flavor fields given by Eq. (2.1) are the solutions of the Euler-Lagrange equation for Eq. (2.2) and therefore can be completely determined in terms of the two free spin-0 fields given by Eq. (2.5) and the mixing angle θ.

This, however, gives rise to a highly nontrivial relationship between the Fock space of free fields and that of interacting fields. To build the Fock space of flavor eigenstates we consider the representation of the transformation consistent with Eq. (2.1) in the Fock space of mass eigenstates. Using the Baker-Hausdorff lemma, we can write the generator of this transformation as

$$\begin{aligned} \hat{S} &= \int d^3x [\dot{\phi}_\alpha^\dagger(x) \phi_\beta(x) + \phi_\beta^\dagger(x) \dot{\phi}_\alpha(x) \\ &\quad - \dot{\phi}_\beta^\dagger(x) \phi_\alpha(x) - \phi_\alpha^\dagger(x) \dot{\phi}_\beta(x)] \\ &= \int d^3x [\dot{\varphi}_1^\dagger(x) \varphi_2(x) + \varphi_2^\dagger(x) \dot{\varphi}_1(x) \\ &\quad - \dot{\varphi}_2^\dagger(x) \varphi_1(x) - \varphi_1^\dagger(x) \dot{\varphi}_2(x)] \end{aligned} \tag{2.7}$$

so that Eq. (2.1) can be written in the form

$$\phi_\alpha = e^{-i\hat{S}\theta} \varphi_1 e^{i\hat{S}\theta},$$

$$\phi_\beta = e^{-i\hat{S}\theta} \varphi_2 e^{i\hat{S}\theta} \tag{2.8}$$

with the transformation operator given by

$$G(\theta) = e^{i\hat{S}\theta}. \tag{2.9}$$

The similarity transformation given by Eq. (2.8) relates the free field operators $\varphi_{1,2}$ to the interacting fields $\phi_{\alpha,\beta}$. These relationships can be obtained from the requirement of the inner product conservation

$$\langle \alpha | \phi | \beta \rangle = \langle a | \varphi | b \rangle, \tag{2.10}$$

where a linear transformation of state vector according to

$$|\alpha\rangle = G^{-1}(\theta) |a\rangle \tag{2.11}$$

relates the two Hilbert spaces, i.e., mass-eigenstate space $\mathcal{H}_{1,2}$ and flavor-eigenstate space $\mathcal{H}_{\alpha,\beta} = G^{-1}(\theta)\mathcal{H}_{1,2}$. The transformation given by Eq. (2.8) can also be viewed as the ''rotation'' of basis in the Hilbert space of quantum states diagonalizing the bilinear Lagrangian given by Eq. (2.2).

The operator $\hat{S}$ can then be written in terms of ladder operators $a_{\vec{k}i}$ and $b_{\vec{k}i}$ as follows:

$$\begin{aligned} \hat{S} = \sum_{\vec{k}} \frac{i}{2} \{ &\gamma_+ (a_{\vec{k}1} a_{\vec{k}2}^\dagger + b_{-\vec{k}1} b_{-\vec{k}2}^\dagger - a_{\vec{k}1}^\dagger a_{\vec{k}2} - b_{-\vec{k}1}^\dagger b_{-\vec{k}2}) \\ &+ \gamma_- (a_{\vec{k}1} b_{-\vec{k}2} + a_{\vec{k}2} b_{-\vec{k}1} - a_{\vec{k}1}^\dagger b_{-\vec{k}2}^\dagger - b_{-\vec{k}1}^\dagger a_{\vec{k}2}^\dagger) \}, \end{aligned} \tag{2.12}$$

where we denote $\gamma_+ = \sqrt{\epsilon_1(k)/\epsilon_2(k)} + \sqrt{\epsilon_2(k)/\epsilon_1(k)}$ and $\gamma_- = \sqrt{\epsilon_1(k)/\epsilon_2(k)} - \sqrt{\epsilon_2(k)/\epsilon_1(k)}$. Here $\gamma_+^2 - \gamma_-^2 = 4$. From Eq. (2.12) we note that Eq. (2.8) makes each cluster $\Omega_{\vec{k}}$, defined by linear superposition of operators $(a_{\vec{k}1}, a_{\vec{k}2}, b_{-\vec{k}1}^\dagger, b_{-\vec{k}2}^\dagger)$, transform into itself, i.e., $e^{-i\hat{S}\theta} \Omega_{\vec{k}} e^{i\hat{S}\theta} = \Omega_{\vec{k}}$. The same can be said about its Hermitian conjugate $\Omega_{\vec{k}}^\dagger$. This means that we can consider the transformation given by Eq. (2.8) within each cluster with a specific momentum $\vec{k}$:

$$\hat{S}_{\vec{k}}=\frac{i}{2}\{\gamma_+(a_{\vec{k}1}a^\dagger_{\vec{k}2}+b_{-\vec{k}1}b^\dagger_{-\vec{k}2}-a^\dagger_{\vec{k}1}a_{\vec{k}2}-b^\dagger_{-\vec{k}1}b_{-\vec{k}2}) + \gamma_-(a_{\vec{k}1}b_{-\vec{k}2}+a_{\vec{k}2}b_{-\vec{k}1}-a^\dagger_{\vec{k}1}b^\dagger_{-\vec{k}2}-b^\dagger_{-\vec{k}1}a^\dagger_{\vec{k}2})\}. \tag{2.13}$$

Thus, the total transformation is given by $e^{i\hat{S}\theta}=\Pi_{\vec{k}}e^{i\hat{S}_{\vec{k}}\theta}$. It is also convenient to express $\hat{S}_{\vec{k}}$ as $\hat{S}_{\vec{k}}\equiv\sqrt{2}(\hat{T}^\dagger_{\vec{k}}+\hat{T}_{\vec{k}})$ with the operator $\hat{T}_{\vec{k}}$ defined by

$$\hat{T}_{\vec{k}}=-\frac{i}{2\sqrt{2}}[\gamma_+(a^\dagger_{\vec{k}1}a_{\vec{k}2}-b_{-\vec{k}1}b^\dagger_{-\vec{k}2}) + \gamma_-(a^\dagger_{\vec{k}1}b^\dagger_{-\vec{k}2}-a_{\vec{k}2}b_{-\vec{k}1})], \tag{2.14}$$

where the commutation relation $[\hat{T}_{\vec{k}},\hat{T}^\dagger_{\vec{k}}]=1$ is satisfied between $\hat{T}_k$ and $\hat{T}^\dagger_k$ just the same way as the particle creation and annihilation operators satisfy the commutation relations. With the operators $\hat{T}_k$ and $\hat{T}^\dagger_k$, we can directly apply Eq. (2.9) to the mass-eigenstate vacuum and obtain

$$|0;\theta,\vec{k}\rangle_{\alpha,\beta}=\sum_{n=0}^{\infty}\frac{(-i\theta\sqrt{2})^n}{n!}\sum_{l=0}^{n}C^l_n(\theta)(\hat{T}^\dagger_{\vec{k}})^l\hat{T}^{n-l}_{\vec{k}}|0\rangle_{1,2}, \tag{2.15}$$

where $C^l_n(\theta)$ are the generalized binomial coefficients that can be found after appropriate orderings of $\hat{T}$ and $\hat{T}^\dagger$ are carried out. In the expression given by Eq. (2.15), one can treat the operators $\hat{T}_{\vec{k}}$ ($\hat{T}^\dagger_{\vec{k}}$) as the annihilation (creation) operator of the vacuum fluctuation.

For simplicity we now suppress the momentum notation in the ladder operators and take the flavor vacuum state in the most general form

$$|0;\theta\rangle=\sum_{n,l,m,k}C_{nlmk}(\theta)(a^\dagger_1)^n(a^\dagger_2)^l(b^\dagger_{-1})^m(b^\dagger_{-2})^k|0\rangle. \tag{2.16}$$

Applying the flavor annihilation operators to this vacuum, we obtain an infinite number of coupled linear equations for the coefficients $C_{nlmk}(\theta)$ and solve these equations in Appendix A. As shown in Appendix A, we find

$$|0,\theta\rangle=\mathcal{Z}\sum_{n,l}\frac{1}{n!l!}(Z_{11}a^\dagger_1b^\dagger_{-1}+Z_{12}a^\dagger_1b^\dagger_{-2})^n \times(-Z_{11}a^\dagger_2b^\dagger_{-2}+Z_{12}a^\dagger_2b^\dagger_{-1})^l|0\rangle, \tag{2.17}$$

where $\mathcal{Z}=\langle 0|0,\theta\rangle$ is the normalization factor to be fixed by $\langle\theta;0|0;\theta\rangle=1$ and the coefficients Z_{11} and Z_{12} are given by

$$Z_{11}=\frac{\gamma_+\gamma_-\sin^2\theta}{4\left(\cos^2\theta+\frac{\gamma_+^2}{4}\sin^2\theta\right)}=\frac{\gamma_+\gamma_-\sin^2\theta}{4\left(1+\frac{\gamma_-^2}{4}\sin^2\theta\right)} \tag{2.18}$$

$$Z_{12}=\frac{-\gamma_-\sin 2\theta}{4\left(\cos^2\theta+\frac{\gamma_+^2}{4}\sin^2\theta\right)}=\frac{-\gamma_-\sin 2\theta}{4\left(1+\frac{\gamma_-^2}{4}\sin^2\theta\right)}.$$

Here, we note that the coefficients Z_{11} and Z_{12} [see Eq. (2.18)] can be written as

$$Z_{11}=\chi\cdot x,\quad Z_{12}=\chi\cdot y, \tag{2.19}$$

where

$$\chi=\frac{\gamma_-\sin\theta}{2\sqrt{1+\frac{\gamma_-^2\sin^2\theta}{4}}}$$

$$x=\frac{\gamma_+\sin\theta}{2\sqrt{\cos^2\theta+\frac{\gamma_+^2\sin^2\theta}{4}}}$$

$$y=\frac{-\cos\theta}{\sqrt{\cos^2\theta+\frac{\gamma_+^2\sin^2\theta}{4}}}. \tag{2.20}$$

Thus, the flavor vacuum state given by Eq. (2.17) can be rewritten as

$$|0,\theta\rangle=\mathcal{Z}\sum_{n,l}\frac{\chi^{n+l}}{n!l!}(xa^\dagger_1b^\dagger_{-1}+ya^\dagger_1b^\dagger_{-2})^n \times(-xa^\dagger_2b^\dagger_{-2}+ya^\dagger_2b^\dagger_{-1})^l|0\rangle. \tag{2.21}$$

This result can be further reduced as

$$|0,\theta\rangle=\mathcal{Z}\sum_{n,l}\frac{\chi^{n+l}}{n!l!}(a^\dagger_1c^\dagger_1)^n(a^\dagger_2c^\dagger_2)^l|0\rangle, \tag{2.22}$$

by defining new ladder operators

$$c_1=xb_{-1}+yb_{-2},$$

$$c_2=-xb_{-2}+yb_{-1}, \tag{2.23}$$

where it is easy to check that $[c_1,c^\dagger_1]=x^2+y^2=1$, $[c_2,c^\dagger_2]=1$, $[c_1,c^\dagger_2]=0$. Now, it is possible to compute directly the value of $\mathcal{Z}$, because

$$\langle 0,\theta|0,\theta\rangle=\mathcal{Z}^2\sum_{n,l}\frac{\chi^{2(n+l)}}{n!^2l!^2}n!^2l!^2=\mathcal{Z}^2\left(\sum_n\chi^{2n}\right)^2=\frac{\mathcal{Z}^2}{(1-\chi^2)^2} \tag{2.24}$$

and the flavor vacuum is normalized to be 1, i.e., $\langle 0,\theta|0,\theta\rangle = 1$. Thus, we find

$$\mathcal{Z} = 1-\chi^2 = \frac{1}{1+\gamma_-^2 \sin^2\theta/4}. \tag{2.25}$$

We see that the flavor vacuum state indeed exists in the Fock space of mass eigenstates and the normalization factor $\mathcal{Z}$ is finite but less than 1 for any value of $\gamma_- \sin\theta$ in the exact vacuum treatment. The same has been obtained in Ref. [11] solving a differential equation of $\langle 0|0,\theta\rangle = \langle 0|G(\theta)|0\rangle$. While such method of derivation using a differential equation has been known for some time [8,10–12], our algebraic method presented in this work is a new development.

This proves then the unitary inequivalence between the two Fock spaces of mass and flavor in the infinite volume limit following the procedure discussed in Ref. [11]:

$$\lim_{V\to\infty} \langle 0|0,\theta\rangle_{\alpha,\beta} = \lim_{V\to\infty} \exp\left(\frac{V}{2\pi^3}\int d^3k \ln \mathcal{Z}\right) = 0, \tag{2.26}$$

for any time. While we agree with Ref. [11] on the point that only the infinite volume limit can warrant the unitary inequivalence even in the boson case, we note that the perturbative expansion of the exact vacuum in the boson case is dramatically different from the case of the fermion. Although the normalization factor $\mathcal{Z}$ is a finite function for all values of $\gamma_- \sin\theta$, we observe that this expression given by Eq. (2.25) has singularity on the complex plane at $\gamma_- \sin\theta = 2i$. This is in remarkable contrast from the fermion case where the corresponding result $\mathcal{Z}_{fermion} = 1-\gamma_-^2 \sin^2\theta/4$ does not have any singularity on the complex plane. Thus, the flavor vacuum $|0,\theta\rangle$ in terms of series in θ shall have a critical point and this would result in the divergence of the Taylor series expansion for $\langle 0|0,\theta\rangle$ in powers of θ because such expansion only makes sense for small θ values. As we explicitly show in Appendix B, the series given by Eq. (2.15) is indeed divergent in the region $\gamma_- \theta > 2$. Such divergence does not occur in the fermion case. We also present some clarifying remarks in Appendix C regarding the Green function method and the aribitrary mass parametrization discussed in the previous literature [10,11,13].

III. LADDER OPERATORS AND CONDENSATIONS

In the previous section, we have built the representation of the mixing transformation given by Eq. (2.1) in the operator space of $\varphi_{1,2}$, where the action of mixing is given by the similarity transformation given by Eq. (2.8). We also considered the representation defined by operating $G^{-1}(\theta)$ in the Fock space and showed the unitary inequivalence between the two (mixed and unmixed) Fock spaces in the infinite volume limit.

Let us now further investigate these representations to come up with physically measurable quantities. The fields $\varphi_{1,2}$ are defined by a superposition of ladder operators $a_{1,2}$ and $b_{1,2}$ that form the basis in a linear Hilbert space of mass eigenstate fields. Using Eqs. (2.5) and (2.8), one can immediately obtain annihilation operators for the mixed (flavor) fields that are consistent with the Pontecorvo mixing relationship,

$$a_{\alpha,\beta} = G^{-1}(\theta) a_{1,2} G(\theta), \tag{3.1}$$

$$b_{\alpha,\beta} = G^{-1}(\theta) b_{1,2} G(\theta).$$

This is also consistent with the definition of flavor vacuum as the lowest energy state, i.e.,

$$\langle 0,\theta|\hat{H}(\theta)|0,\theta\rangle = \langle 0|G(\theta)\hat{H}(\theta)G^{-1}(\theta)|0\rangle = \langle 0|\hat{H}_0|0\rangle = 0, \tag{3.2}$$

where $\hat{H}(\theta)$ and $\hat{H}_0$ are the Hamiltonians of mixed fields and unmixed fields, respectively. Straightforward application of the Baker-Hausdorff lemma to Eq. (3.1) yields

$$\begin{aligned}
a_\alpha &= a_1 \cos\theta + \frac{\sin\theta}{2}(\gamma_+ a_2 + \gamma_- b_{-2}^\dagger),\\
a_\beta &= a_2 \cos\theta + \frac{\sin\theta}{2}(-\gamma_+ a_1 + \gamma_- b_{-1}^\dagger),\\
b_{-\alpha} &= b_{-1} \cos\theta + \frac{\sin\theta}{2}(\gamma_+ b_{-2} + \gamma_- a_2^\dagger),\\
b_{-\beta} &= b_{-2} \cos\theta + \frac{\sin\theta}{2}(-\gamma_+ b_{-1} + \gamma_- a_1^\dagger).
\end{aligned} \tag{3.3}$$

It is also not difficult to reverse Eq. (3.3) in order to obtain how the mass-eigenstate ladder operators are expressed in terms of flavor ones. Using the above relationships we can also find the time dependence of the flavor-eigenstate ladder operators in the Heisenberg picture since the time evolution of mass-eigenstate ladder operators are given by

$$\begin{aligned}
a_{1,2}(t) &= e^{i\hat{H}_0 t} a_{1,2} e^{-i\hat{H}_0 t} = e^{-i\epsilon_{1,2} t} a_{1,2},\\
b_{1,2}(t) &= e^{i\hat{H}_0 t} b_{1,2} e^{-i\hat{H}_0 t} = e^{-i\epsilon_{1,2} t} b_{1,2}.
\end{aligned} \tag{3.4}$$

In particular, after introducing more compact notation

$$C = \cos\theta, \quad S_+ = \frac{\sin\theta\gamma_+}{2}, \quad S_- = \frac{\sin\theta\gamma_-}{2}, \tag{3.5}$$

we find

$$\begin{aligned}
a_{\alpha t} = {} & (C^2 e^{-i\epsilon_1 t} + S_+^2 e^{-i\epsilon_2 t} - S_-^2 e^{i\epsilon_2 t}) a_\alpha \\
& + C S_+ (e^{-i\epsilon_2 t} - e^{-i\epsilon_1 t}) a_\beta + S_+ S_- (e^{i\epsilon_2 t} - e^{-i\epsilon_2 t}) b_{-\alpha}^\dagger \\
& + C S_- (e^{i\epsilon_2 t} - e^{-i\epsilon_1 t}) b_{-\beta}^\dagger,
\end{aligned} \tag{3.6}$$

$$\begin{aligned}
a_{\beta t} = {} & (C^2 e^{-i\epsilon_2 t} + S_+^2 e^{-i\epsilon_1 t} - S_-^2 e^{i\epsilon_1 t}) a_\beta \\
& + C S_+ (e^{-i\epsilon_2 t} - e^{-i\epsilon_1 t}) a_\alpha + S_+ S_- (e^{-i\epsilon_1 t} - e^{i\epsilon_1 t}) b_{-\beta}^\dagger \\
& + C S_- (e^{i\epsilon_1 t} - e^{-i\epsilon_2 t}) b_{-\alpha}^\dagger,
\end{aligned}$$

$$
\begin{aligned}
b_{-\alpha t} &= (C^2 e^{-i\epsilon_1 t} + S_+^2 e^{-i\epsilon_2 t} - S_-^2 e^{i\epsilon_2 t}) b_{-\alpha} \\
&\quad + CS_+ (e^{-i\epsilon_2 t} - e^{-i\epsilon_1 t}) b_{-\beta} + S_+ S_- (e^{i\epsilon_2 t} - e^{-i\epsilon_2 t}) a_\alpha^\dagger \\
&\quad + CS_- (e^{i\epsilon_2 t} - e^{-i\epsilon_1 t}) a_\beta^\dagger , \\
b_{-\beta t} &= (C^2 e^{-i\epsilon_2 t} + S_+^2 e^{-i\epsilon_1 t} - S_-^2 e^{i\epsilon_1 t}) b_{-\beta} \\
&\quad + CS_+ (e^{-i\epsilon_2 t} - e^{-i\epsilon_1 t}) b_{-\alpha} + S_+ S_- (e^{-i\epsilon_1 t} - e^{i\epsilon_1 t}) a_\beta^\dagger \\
&\quad + CS_- (e^{i\epsilon_1 t} - e^{-i\epsilon_2 t}) a_\alpha^\dagger ,
\end{aligned}
$$

from which we can also obtain the *unequal*-time commutation relationships:

$$
\begin{aligned}
[a_\alpha, a_{\alpha t}^\dagger] &= [b_{-\alpha}, b_{-\alpha t}^\dagger] \\
&= C^2 e^{i\epsilon_1 t} + S_+^2 e^{i\epsilon_2 t} - S_-^2 e^{-i\epsilon_2 t} = A_{\alpha\alpha} , \\
[a_\beta, a_{\beta t}^\dagger] &= [b_{-\beta}, b_{-\beta t}^\dagger] \\
&= C^2 e^{i\epsilon_2 t} + S_+^2 e^{i\epsilon_1 t} - S_-^2 e^{-i\epsilon_1 t} = A_{\beta\beta} , \\
[a_\beta, a_{\alpha t}^\dagger] &= [a_\alpha, a_{\beta t}^\dagger] \\
&= [b_{-\beta}, b_{-\alpha t}^\dagger] = [b_{-\alpha}, b_{-\beta t}^\dagger] \\
&= CS_+ (e^{i\epsilon_2 t} - e^{i\epsilon_1 t}) = A_{\beta\alpha} , \\
[b_{-\beta}, a_{\alpha t}] &= [a_\beta, b_{-\alpha t}] \\
&= -[b_{-\alpha}, a_{\beta t}]^* = -[a_\alpha, b_{-\beta t}]^* \\
&= CS_- (e^{i\epsilon_2 t} - e^{-i\epsilon_1 t}) = A_{\bar{\beta}\alpha}^- , \\
[b_{-\alpha}, a_{\alpha t}] &= [a_\alpha, b_{-\alpha t}] = S_+ S_- (e^{i\epsilon_2 t} - e^{-i\epsilon_2 t}) = A_{\alpha\alpha}^- , \\
[b_{-\beta}, a_{\beta t}] &= [a_\beta, b_{-\beta t}] = S_+ S_- (e^{-i\epsilon_1 t} - e^{i\epsilon_1 t}) = A_{\bar{\beta}\beta}^- .
\end{aligned}
\tag{3.7}
$$

All other commutators are either zeros or can be expressed in terms of the above ones. Equations (3.3), (3.6), (3.7) in fact define all the dynamics of Pontecorvo mixing for two quantum fields. To show how these relationships can be used to calculate the dynamical parameters of the mixed (interacting) fields, one can consider the time evolution of cluster $\Omega_{\vec{k}}$ defined in Sec. II. As discussed in Sec. II, however, this cluster is invariant under the $G^{-1}(\theta)$ transformation. Thus, we can consider $\Omega_{\vec{k}}$ with a particular $\vec{k}$ independently from all other momentum values.

We now calculate the number of particles with a definite mass condensed in the flavor vacuum state $|0'\rangle = |0\rangle_{\alpha,\beta}$. Let us consider the condensation of the particle with a definite mass, for example, $Z_1 = \langle 0'|N_1|0'\rangle$. Using the inverse relation of Eq. (3.3):

$$
a_1 = a_\alpha \cos\theta - \frac{\sin\theta}{2}(\gamma_+ a_\beta + \gamma_- b_{-\beta}^\dagger), \tag{3.8}
$$

we can get

$$
Z_1 = \langle 0'|a_1^\dagger a_1|0'\rangle = \frac{\sin^2\theta\gamma_-^2}{4}\langle 0'|b_{-\beta}b_{-\beta}^\dagger|0'\rangle . \tag{3.9}
$$

One can show that the same result is true for $Z_2 = \langle 0'|N_2|0'\rangle$. Thus, the condensate density of particles with a definite mass in the flavor vacuum is given by

$$
Z_1 = Z_2 = S_-^2 = \frac{\sin^2\theta\gamma_-^2}{4} . \tag{3.10}
$$

Apparently the condensate densities for particles with definite flavor in the mass vacuum, i.e., $\langle 0|N_{\alpha(\beta)}|0\rangle$, are also given by S_-^2. Let us now consider the number of particles with a definite flavor in the flavor vacuum, for example, $Z_\alpha(t) = \langle 0'|N_\alpha(t)|0'\rangle$. Using Eq. (3.6), one can easily show that

$$
\begin{aligned}
Z_\alpha(t) = \langle 0'| &[S_+ S_- (e^{i\epsilon_2 t} - e^{-i\epsilon_2 t}) b_{-\alpha}^\dagger \\
&+ CS_- (e^{i\epsilon_2 t} - e^{-i\epsilon_1 t}) b_{-\beta}^\dagger]^\dagger \\
&\times [S_+ S_- (e^{i\epsilon_2 t} - e^{-i\epsilon_2 t}) b_{-\alpha}^\dagger \\
&+ CS_- (e^{i\epsilon_2 t} - e^{-i\epsilon_1 t}) b_{-\beta}^\dagger] |0'\rangle
\end{aligned}
\tag{3.11}
$$

and thus

$$
Z_\alpha(t) = 4S_-^2 S_+^2 \sin^2(\epsilon_2 t) + 4S_-^2 C^2 \sin^2\left(\frac{\epsilon_1 + \epsilon_2}{2} t\right). \tag{3.12}
$$

Similarly, we get for the β particles

$$
Z_\beta(t) = 4S_-^2 S_+^2 \sin^2(\epsilon_1 t) + 4S_-^2 C^2 \sin^2\left(\frac{\epsilon_1 + \epsilon_2}{2} t\right). \tag{3.13}
$$

We see that the number of particles with a definite flavor in the flavor vacuum is indeed not zero. This is due to the fact that the flavor vacuum is not an energy eigenstate of the Hamiltonian $\hat{H}(\theta)$ and changes with the time translation producing and destroying coherently virtual particle/antiparticle pairs. It shows a significant difference from the ordinary quantum mechanical treatment without considering the vacuum effect, which yields $Z_{\alpha(\beta)} = 0$ for any time. We emphasize that our flavor vacuum here is not perturbative but exact. This is different from the approach, where mass eigenstate vacuum $|0\rangle_{1,2}$ is used instead of the flavor vacuum to generate a flavor eigenstate, e.g., $|\alpha\rangle = a_\alpha^\dagger|0\rangle_{1,2}$. If the flavor vacuum $|0'\rangle$ were replaced by the mass vacuum $|0\rangle_{1,2}$, then we would have obtained $Z_1 = Z_2 = 0$ instead of Eq. (3.10). As discussed above in the exact vacuum treatment, the mass eigenstate vacuum is not annihilated by $a_{\alpha,\beta}$ operators. Indeed the term proportional to $O(\gamma_-)$ remains in the creation/annihilation operators, so that the accuracy in the order of $O(\gamma_-^2)$ can be expected from the results of the exact vacuum approach compared to the perturbative vacuum approximation. The densities of vacuum condensation for antiparticles

$Z_{\bar{1},\bar{2}}, Z_{\bar{\alpha},\bar{\beta}}$ are obtained the same as the densities for the corresponding particles, i.e. $Z_{1,2}, Z_{\alpha,\beta}$ given by Eqs. (3.10), (3.12), and (3.13).

We now consider the flavor oscillations in time for a single particle with flavor α and momentum $\vec{k}$. In the Heisenberg picture, the average number of particles with flavor $a=\alpha$ or β in the flavor state $|\alpha\rangle = a_\alpha^\dagger |0'\rangle$ is given by

$$\langle N_a(t)\rangle = \langle \alpha | N_a(t) | \alpha \rangle = \langle 0' | a_\alpha a_{at}^\dagger a_{at} a_\alpha^\dagger | 0' \rangle. \quad (3.14)$$

In Eq. (3.14), we note that we use the flavor vacuum to obtain the exact result for the flavor oscillations. Later, in Sec. IV, we numerically compare the exact result with the previous approximate result [9]. Using Eqs. (3.3), (3.6), (3.7), we directly apply the standard quantum field theoretic method. Since the flavor vacuum is annihilated by $a_{\alpha,\beta}$, we move a_α in Eq. (3.14) to the right most position and $a_\alpha^\dagger$ to the left most position to annihilate the flavor vacuum. What is left is uniquely determined by the unequal time commutation relations given by Eq. (3.7) and we find

$$\begin{aligned}
\langle \alpha | N_{\alpha t} | \alpha \rangle &= \langle 0' | a_{\alpha t}^\dagger a_{\alpha t} | 0' \rangle + |[a_\alpha, a_{\alpha t}^\dagger]|^2 = Z_\alpha + |A_{\alpha\alpha}|^2, \\
\langle \alpha | N_{-\bar{\alpha} t} | \alpha \rangle &= \langle 0' | b_{-\alpha t}^\dagger b_{-\alpha t} | 0' \rangle + |[a_\alpha, b_{-\alpha t}]|^2 \\
&= Z_\alpha + |A_{\alpha\bar{\alpha}}|^2, \\
\langle \alpha | N_{\beta t} | \alpha > &= \langle 0' | a_{\beta t}^\dagger a_{\beta t} | 0' \rangle + |[a_\alpha, a_{\beta t}^\dagger]|^2 = Z_\beta + |A_{\beta\alpha}|^2, \\
\langle \alpha | N_{-\bar{\beta} t} | \alpha \rangle &= \langle 0' | b_{-\beta t}^\dagger b_{-\beta t} | 0' \rangle + |[a_\alpha, b_{-\beta t}]|^2 \\
&= Z_\beta + |A_{\beta\bar{\alpha}}|^2. \quad (3.15)
\end{aligned}$$

Using the notation of C, $S_\pm$, our results are summarized as

$$\begin{aligned}
\langle \alpha | N_{\alpha t} | \alpha \rangle &= 1 + 8C^2 S_-^2 \sin^2\left(\frac{\epsilon_1+\epsilon_2}{2} t\right) + 8 S_-^2 S_+^2 \sin^2(\epsilon_2 t) \\
&\quad - 4C^2 S_+^2 \sin^2\left(\frac{\epsilon_1-\epsilon_2}{2} t\right), \\
\langle \alpha | N_{\beta t} | \alpha \rangle &= 4C^2 S_-^2 \sin^2\left(\frac{\epsilon_1+\epsilon_2}{2} t\right) + 4 S_-^2 S_+^2 \sin^2(\epsilon_1 t) \\
&\quad + 4C^2 S_+^2 \sin^2\left(\frac{\epsilon_1-\epsilon_2}{2} t\right), \\
\langle \alpha | N_{-\bar{\alpha} t} | \alpha \rangle &= 4C^2 S_-^2 \sin^2\left(\frac{\epsilon_1+\epsilon_2}{2} t\right) + 8 S_-^2 S_+^2 \sin^2(\epsilon_2 t), \\
\langle \alpha | N_{-\bar{\beta} t} | \alpha \rangle &= 8C^2 S_-^2 \sin^2\left(\frac{\epsilon_1+\epsilon_2}{2} t\right) + 4 S_-^2 S_+^2 \sin^2(\epsilon_1 t). \quad (3.16)
\end{aligned}$$

As shown in Eq. (3.16), the time dependence of the average number of particles with a definite flavor is rather complicated. It contains oscillating contributions from both the $\alpha \to \beta$ conversion and from the virtual pair creation in a dynamically ''rotating'' flavor vacuum. As discussed in Ref. [9], the $\alpha \to \beta$ conversion process generates the term proportional to

$$\sin^2\left(\frac{\epsilon_1-\epsilon_2}{2} t\right).$$

The terms involving $\epsilon_1+\epsilon_2, \epsilon_1, \epsilon_2$ frequencies in Eq. (3.16) are, however, related to the creation of virtual pairs. For example, the virtual pair creation violates energy conservation within the uncertainty time, i.e., $\Delta E \Delta t \approx 1$ (in our units $\hbar = 1$) and thus both creation and annihilation of, let us say, $(\alpha + \bar{\beta})$ virtual pair must occur within $\tau \approx 1/(\epsilon_1+\epsilon_2)$ time interval. Thus, the terms in Eq. (3.16) involving $\epsilon_1 + \epsilon_2, \epsilon_1, \epsilon_2$ frequencies can be related to the creation of different types of virtual pairs, while the terms involving $\epsilon_1 - \epsilon_2$ are related to the actual $\alpha \to \beta$ conversion.

Using Eq. (3.16), we can also calculate the expectation value of the flavor charge operator defined by $Q_{\alpha,\beta} = N_{\alpha,\beta} - N_{-\bar{\alpha},-\bar{\beta}}$,

$$\begin{aligned}
\langle Q_\alpha \rangle &= 1 - 4C^2 S_+^2 \sin^2\left(\frac{\epsilon_1-\epsilon_2}{2} t\right) + 4CS_-^2 \sin^2\left(\frac{\epsilon_1+\epsilon_2}{2} t\right), \quad (3.17) \\
\langle Q_\beta \rangle &= 4C^2 S_+^2 \sin^2\left(\frac{\epsilon_1-\epsilon_2}{2} t\right) - 4C^2 S_-^2 \sin^2\left(\frac{\epsilon_1+\epsilon_2}{2} t\right),
\end{aligned}$$

or with the conventional parameters,

$$\begin{aligned}
\langle Q_\alpha \rangle &= 1 - \gamma_+^2 \sin^2(2\theta) \sin^2\left(\frac{\epsilon_1-\epsilon_2}{2} t\right) \\
&\quad + \gamma_-^2 \sin^2(2\theta) \sin^2\left(\frac{\epsilon_1+\epsilon_2}{2} t\right), \\
\langle Q_\beta \rangle &= \gamma_+^2 \sin^2(2\theta) \sin^2\left(\frac{\epsilon_1-\epsilon_2}{2} t\right) \\
&\quad - \gamma_-^2 \sin^2(2\theta) \sin^2\left(\frac{\epsilon_1+\epsilon_2}{2} t\right). \quad (3.18)
\end{aligned}$$

From this result, one can also see that there is an additional term proportional to $\sin^2[(\epsilon_1+\epsilon_2)/2t]$ to the usual Pontecorvo formula. As discussed above, the origin of this term can be understood as a contribution from the virtual pair creation in ''rotating'' vacuum. The correction term is of the order of $O(\gamma_-^2)$. As noted earlier, this may explain why it has been found neither in an ordinary quantum mechanical treatment nor in the approximate QFT treatment based on a perturbative vacuum.

We also calculate the time evolution of the coherent state for the two mixed quantum fields. The coherent state has the form

$$|C\alpha\rangle = e^{Ca_\alpha^\dagger} |0'\rangle. \quad (3.19)$$

Extending the above calculation for a single particle, it is not so difficult to verify that the state containing n particles with flavor α can be given by

$$\langle n | N_{\alpha t} | n \rangle = \frac{1}{n!} \langle 0' | a_\alpha^n N_{\alpha t} (a_\alpha^\dagger)^n | 0' \rangle = \langle N_{\alpha t} \rangle + n |A_{\alpha\alpha}|^2. \quad (3.20)$$

Besides $n|A_{\alpha\alpha}|^2$ which is simply n times the probability of $\alpha\to\alpha$ transition, we see in Eq. (3.20) that the condensate contribution is present adding the density of α particles from ''rotating'' vacuum. Applying this result directly to the coherent state expansion, we obtain the following expectation values of the number operator $N_{(\alpha,\beta)t}$ in the coherent state $|C\alpha\rangle$:

$$\langle C\alpha|N_{\alpha t}|C\alpha\rangle = Z_\alpha + |C|^2|A_{\alpha\alpha}|^2,$$
$$\langle C\alpha|N_{\beta t}|C\alpha\rangle = Z_\beta + |C|^2|A_{\beta\alpha}|^2. \tag{3.21}$$

Thus, the expectation values of the flavor charge operator $Q_{(\alpha,\beta)}=N_{(\alpha,\beta)}-N_{(-\bar{\alpha},-\bar{\beta})}$ turn out to be

$$\begin{aligned}\langle C\alpha|Q_\alpha|C\alpha\rangle &= |C|^2\langle Q_\alpha\rangle \\ &= |C|^2\left[1-\gamma_+^2\sin^2(2\theta)\sin^2\left(\frac{\epsilon_1-\epsilon_2}{2}t\right)\right. \\ &\quad \left.+\gamma_-^2\sin^2(2\theta)\sin^2\left(\frac{\epsilon_1+\epsilon_2}{2}t\right)\right], \\ \langle C\alpha|Q_\beta|C\alpha\rangle &= |C|^2\langle Q_\beta\rangle \\ &= |C|^2\left[\gamma_+^2\sin^2(2\theta)\sin^2\left(\frac{\epsilon_1-\epsilon_2}{2}t\right)\right. \\ &\quad \left.-\gamma_-^2\sin^2(2\theta)\sin^2\left(\frac{\epsilon_1+\epsilon_2}{2}t\right)\right].\end{aligned} \tag{3.22}$$

As we can see in Eq. (3.22), the vacuum contributions $Z_{(\alpha,\beta)}$ are removed from the flavor charge expectation values and the results for the coherent state are simply $|C|^2$ times the expectation values of the flavor charge for the single particle state.

IV. APPLICATION TO REAL MESON STATES

We now apply the results for time evolution of two mixing boson fields to the analysis of the η-η' mixing system. The masses are taken to be 549 MeV and 958 MeV, respectively, and of course in the particle rest frame the energies in our formulas reduce to the masses. The phenomenologically allowed mixing angle ($\theta_{SU(3)}$) range of the $\eta\eta'$ system is given between $-10°$ and $-23°$ [15], where the mixing angle $\theta_{SU(3)}$ is defined by Eq. (36) of Ref. [16]. This angle represents the breaking of the SU(3) symmetry, the eigenstates of which are already rotated $-35.26°$ from $u\bar{u}+d\bar{d}$ and $s\bar{s}$ to $\alpha=u\bar{u}+d\bar{d}-2s\bar{s}$ and $\beta=u\bar{u}+d\bar{d}+s\bar{s}$. Thus, our mixing angle is defined by $\theta=\theta_{SU(3)}-35.26°$. Recent analysis of the $\eta\eta'$ mixing angle using a constituent quark model based on the Fock states quantized on the light front can be found in Ref. [16] and the references therein. The optimal value found for $\theta_{\mathrm{SU(3)}}$ was around $-19°$ and thus $\theta\approx-54°$. We use these values in Eqs. (3.16) and (3.17) [or equivalently (3.18)] to determine the evolution of definite flavor particle number and charge.

In Fig. 1, we present both $\langle\alpha|N_{\alpha t}|\alpha\rangle$ (thick solid line) and $\langle Q_\alpha\rangle$ (dotted line) as a function of time when the particle momentum is given by $k=0.1$ GeV. For a comparison, we also show the previous approximate result (thin solid line) based on the perturbative vacuum [9] corresponding to these quantities noting that $\langle\alpha|N_{\alpha t}|\alpha\rangle$ and $\langle Q_\alpha\rangle$ coincide with each other in this approximation as one can see in Eqs. (3.16) and (3.17). As we show in Fig. 1, the population density $\langle\alpha|N_{\alpha t}|\alpha\rangle$ (thick solid line) is completely distorted due to the interaction with the nonperturbative vacuum while the sinusoidal Pontecorvo result (thin solid line) is obtained for the approximate perturbative vacuum treatment. We see the large deviation up to 40% in $\langle\alpha|N_{\alpha t}|\alpha\rangle$. However, one cannot see the same level of deviation in $\langle Q_\alpha\rangle$ and the previous result [9] based on the perturbative vacuum seems to be a good approximation for the description of flavor charge oscillations modulo the accuracy of order $O(\gamma_-^2)$.

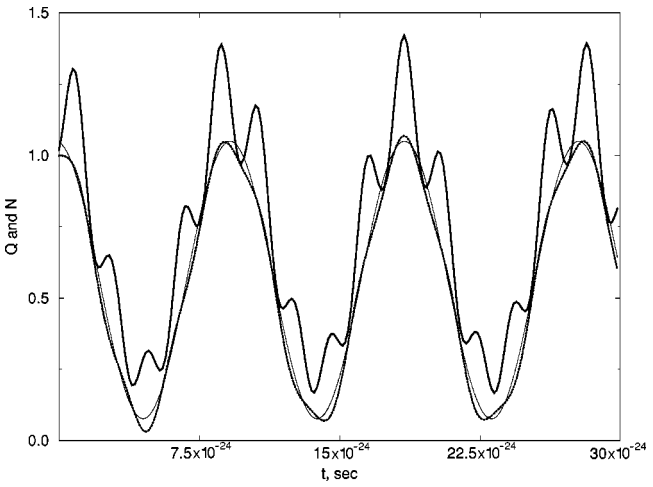

FIG. 1. Comparing population density evolution for $k=0.1$ GeV.

More details of our results on the time evolution of the particle number with the momentum $k=0.1$ GeV are shown in Fig. 2, where the thick solid and dashed lines are $\langle\alpha|N_{\alpha t}|\alpha\rangle$ and $\langle\alpha|N_{\beta t}|\alpha\rangle$, respectively, and the thin solid and dotted lines are respectively the antiparticle contributions of $\langle\alpha|N_{\bar{\alpha}t}|\alpha\rangle$ and $\langle\alpha|N_{\bar{\beta}t}|\alpha\rangle$. The $\eta\eta'$ is one of the

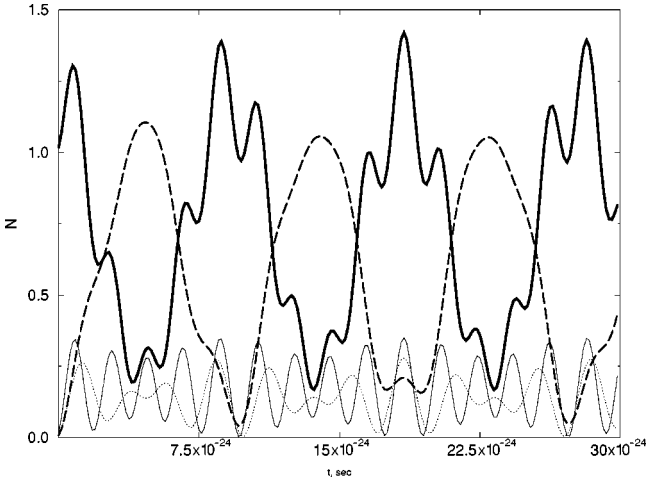

FIG. 2. Population density evolution for $k=0.1$ GeV.

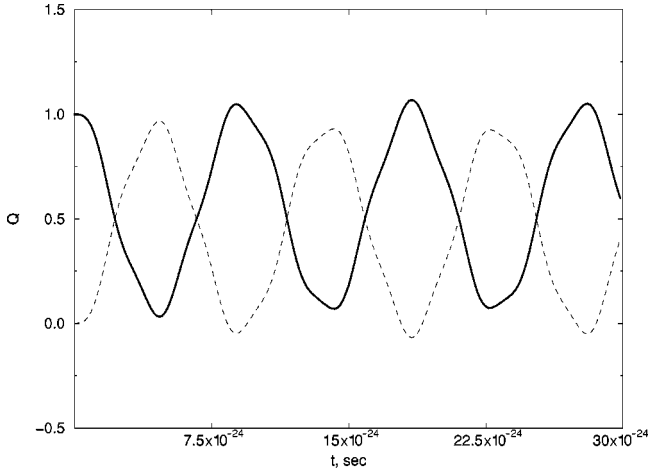

FIG. 3. Flavor charge oscillations.

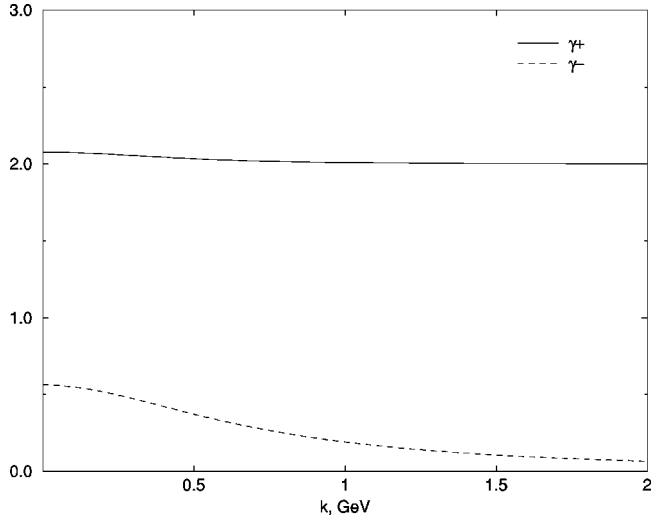

FIG. 4. Mixing amplitudes.

most severely mixed systems due to the great difference in masses of mixed particles. As we have stated earlier, the simple harmonic structure of average particle number usually obtained in quantum mechanics or in an approximate QFT treatment [9] is completely altered as a result of nontrivial interaction with the complicated vacuum. What we see is the superposition of two different cycles as described by Eq. (3.16). From the initial moment of time the population of both α particles (thick solid line) and β particles (thick dashed line) increases. Although the increase of number of β particles in the system is well understood due to $\alpha \to \beta$ conversion, the initial increase of the α population is quite unexpected and caused by α-$\bar{\alpha}$ production from vacuum. The contribution from this process, however, is rather fast so that the general tendency of exchanging α and β particle states can also be seen quite well. In Fig. 2, we also see the oscillations of the antiparticle number in the system. This effect is given in the order of γ_-^2 and usually is absent in an approximate QFT treatment. This is entirely a QFT effect which cannot be obtained within the framework of quantum mechanics. In QFT, besides the beams of α and β particles moving in the $\vec{k}$ direction, we necessarily have an antiparticle beam traveling in the opposite direction. The population density in this beam is correlated with a particle beam so that the total flavor is preserved. The existence of the beam is caused by ''dynamically rotating'' vacuum disturbance at the initial time of the α particle emergence. One should also note that the existence of ''recoil'' antiparticle beam is preserved in the more general wave-packet QFT treatment of the mixing problem. Thus, the mixed particle of definite flavor not only produces the usual oscillation of population density in time (or space [17]) but also is accompanied by emitting the beam of antiparticles traveling in the direction opposite to the beam of particles. These effects are in principle testable in the experiments.

In Fig. 3, we also plot more details on the time dependence of flavor charge expectation value with the same momentum $k=0.1$ GeV. The thick solid and dashed lines are $\langle Q_\alpha \rangle$ and $\langle Q_\beta \rangle$, respectively. One can see that they exhibit mainly the simple periodic structure similar to the approximate QFT results [8,9] and only slightly distorted due to interaction with the vacuum. The amount of distortion is of γ_-^2 order, i.e., about 10% for this case. An interesting feature is, however, the presence of regions where the flavor charge of a given sort of particles changes sign, which means that antiparticles outnumber the particles. The process can be physically understood as a result of α-$\bar{\alpha}$ production when the number of α particles is small due to the $\alpha \to \beta$ transition.

It is also interesting and experimentally testable that the efficiency of conversion processes and the flavor-vacuum disturbance depend essentially on the energy of the original particle. The dependence is effective to the relativistic mass of particles so that the QFT-mixing effects are decreasing with the energy increase of the flavor particle. The distribution of intensity for simple quantum mechanical mixing and QFT mixing is given by the relationship of amplitudes $\gamma_+(k)$, $\gamma_-(k)$ which determine the intensity of a_2 and $b_{-2}^\dagger$ terms in a_α [see Eq. (3.3)]. In Fig. 4, we plot their dependence on the momentum of the emitted α particle. As we can see in Fig. 4, γ_+ amplitude decreases as k increases and goes to 2 as $k \to \infty$. In this limit, γ_+ defines mixing due to a simple rotation between a_1 and a_2 states. Since it can be successfully computed within the framework of quantum mechanics, it gives the usual Pontecorvo formula with only one oscillatory term. On the other hand, γ_- appears with an antiparticle creation operator and describes the Bogoliubov rotation between a_1 and b_{-1} states. This term is also responsible for $(\epsilon_1+\epsilon_2)/2$ high frequency term and antiparticle beam creation. As we see in Fig. 4, it decreases as $k \to \infty$ and the mass difference becomes washed out by the relativistic gain of mass. This also means that at ultrarelativistic limit the QFT-mixing effects vanish so that the simple Pontecorvo formula is restored for flavor oscillation.

To demonstrate the energy dependence, we show in Fig. 5 the plot of population densities evolving with time for the larger momentum $k=0.5$ GeV. The line assignments are the same as shown in Fig. 2. As easily seen in Fig. 5, the intensity of the antiparticle beam decreases dramatically to about 10% [in contrast to (20–40)% in Fig. 2] of initial intensity.

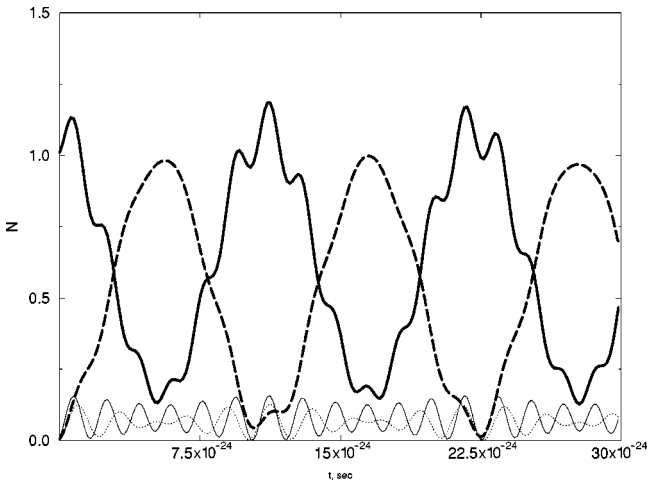

FIG. 5. Population density evolution for $k=0.5$ GeV.

The initial increase in the population density fluctuation in particle beams also reduces even though the quantum mechanical simple oscillations with $(\epsilon_1-\epsilon_2)/2$ frequency are still visibly distorted. Two beams nevertheless demonstrate strong correlation of the same kind as the correlation in the quantum mechanical Einstein-Podolsky-Rosen (EPR) paradox problem so that total flavor charge is preserved as it should be. It is also noticeable that there exist moments of time when the antiparticle outnumbers the particle of the same sort thus producing a negative flavor charge as shown in Fig. 3.

V. CONCLUSIONS AND DISCUSSIONS

The non-trivial scalar and pseudoscalar meson mixing effects may be understood by the condensation of corresponding flavor states in the vacuum [9]. We have now extended the analysis replacing the perturbative vacuum to the nonperturbative (flavor) vacuum. Central to this analysis is the interplay between the base (unmixed) Fock space and the physical Fock space. Their nontrivial relationship gives rise to the mixing and oscillation phenomena. While the similar quantum field theoretic formulation was presented for fermion mixing [8,10], as well as boson mixing [11], our analysis differs in the derivation of the normalization factor $\mathcal{Z}$ given by Eq. (2.25) which is crucial to show the unitary inequivalence between the mass vacuum and the flavor vacuum. We presented a new algebraic method which is distinct from the conventional method of using a differential equation for $\mathcal{Z}$. While the unitary inequivalence occurs only in the infinite volume limit even for the boson case as discussed in Ref. [11], we find an intrinsic difference between the fermion and boson cases. As shown in this work, the normalization factor $\mathcal{Z}$ for the boson given by Eq. (2.25) has a singularity on the complex plane at $\gamma_-\sin\theta=2i$ while the corresponding result for the fermion does not have any singularity. As we summarized in Appendix B, this singularity corresponds to the divergence of the Taylor series expansion in powers of θ for the region $\gamma_-\theta>2$. For both the boson and fermion cases, however, the non-trivial observable mixing phenomena cannot occur unless there is both a nonzero mixing angle and also a nonzero mass (energy) difference between the two physically measurable mixed states. Dramatic small oscillations due to the virtual pair creation occur in the exact vacuum analysis, while only simple sinusoidal Pontecorvo oscillations occur in the perturbative vacuum treatment. Some clarifying remarks on the Green function method and the arbitrary mass parametrization discussed in the previous literature [10,11,13] are also summarized in Appendix C.

As a physical application, we used our formulation to analyze the $\eta\eta'$ system and found that the measured mixing angle and mass difference between η and η' can be related to the non-trivial flavor condensation in the vacuum. However, more fundamental questions such as the translation of the condensation in hadronic degrees of freedom to those in quark and gluon degrees of freedom remain unanswered. The answer to this question depends on the dynamics responsible for the confinement of quark and gluon degrees of freedom and perhaps has to rely on lattice QCD and/or some phenomenological model that accommodates strongly interacting QCD. Further investigation along this line is underway. Also, it would be interesting to look at the mixing transformations between gauge vector bosons governed by the Weinberg angle in the electroweak theory as well as vector mesons such as the ρ and ω. While the statistics are the same as the scalar and pseudoscalar bosons considered here, there will be additional spin dependent interactions which complicate the analysis.

ACKNOWLEDGMENTS

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APPENDIX A: COUPLED EQUATIONS FOR COMPUTING THE FLAVOR VACUUM STRUCTURE

In this appendix, we summarize the procedure for deriving Eq. (2.17) that describes a structure of the flavor vacuum. We first define flavor vacuum in terms of mass eigenstates as the most general linear superposition of the form:

$$|0;\theta\rangle=\sum_{n,l,m,k} C_{nlmk}(\theta)(a_1^\dagger)^n(a_2^\dagger)^l(b_{-1}^\dagger)^m(b_{-2}^\dagger)^k|0\rangle$$
$$=\sum_{n,l,m,k}\frac{C'_{nlmk}(\theta)}{n!l!}(a_1^\dagger)^n(a_2^\dagger)^l(b_{-1}^\dagger)^m(b_{-2}^\dagger)^k|0\rangle. \tag{A1}$$

Then, using the definition of flavor vacuum

$$a_{\alpha,\beta}|0;\theta\rangle=0,$$
$$b_{-\alpha,-\beta}|0;\theta\rangle=0, \tag{A2}$$

and the explicit expression for the ladder operators given by Eq. (3.3), one can derive an infinite set of linear equations for C'_{nlkm} coefficients:

$$CC'_{n+1,lmk}+S_+C'_{n,l+1,mk}+S_-C'_{nlm,k-1}=0,$$

$$CC'_{n,l+1,mk}-S_+C'_{n+1,lmk}+S_-C'_{nl,m-1,k}=0,$$

$$n,l,m,k=0,1,2,3,\ldots. \quad \text{(A3)}$$

To solve this infinite set of equations, we can express $C'_{n+1,lmk}, C'_{n,l+1,km}$ in terms of $C'_{nl,m-1,k}, C'_{nlm,k-1}$ so that we step by step reduce the $n+l$ number of particles. Denoting

$$\begin{pmatrix} Z_{12} & Z_{11} \\ Z_{22} & Z_{21} \end{pmatrix} = -S_- \begin{pmatrix} C & S_+ \\ -S_+ & C \end{pmatrix}^{-1} \quad \text{(A4)}$$

(notation for $\hat{Z}$ is chosen in correspondence to the index of particle type), we can write this relationship as

$$C'_{n+1,lmk}=Z_{12}C'_{nlm,k-1}+Z_{11}C'_{nl,m-1,k}$$

$$C'_{n,l+1,mk}=Z_{22}C'_{nlm,k-1}+Z_{21}C'_{nl,m-1,k}. \quad \text{(A5)}$$

One also can write this in a symbolic manner introducing a kind of shifting operator with definition

$$\hat{k}C'_{nlmk}=C'_{nlm,k-1},$$

$$\hat{m}C'_{nlmk}=C'_{nl,m-1,k}. \quad \text{(A6)}$$

With the use of Eq. (A6), Eq. (A5) may be rewritten as

$$C'_{n+1,lmk}=(Z_{12}\hat{k}+Z_{11}\hat{m})C'_{nlmk}$$
$$=(Z_{12}\hat{k}+Z_{11}\hat{m})^2C'_{n-1,lmk}=\ldots,$$
$$C'_{n,l+1,mk}=(Z_{22}\hat{k}+Z_{21}\hat{m})C'_{nlmk}$$
$$=(Z_{22}\hat{k}+Z_{21}\hat{m})^2C'_{n,l-1,mk}=\ldots, \quad \text{(A7)}$$

and finally it can be written as

$$C'_{nlmk}=(Z_{12}\hat{k}+Z_{11}\hat{m})^n(Z_{22}\hat{k}+Z_{21}\hat{m})^lC'_{00mk}=\left(\sum_{m'=0}^{n}\sum_{t'=0}^{l}C_n^{m'}C_l^{t'}Z_{11}^{m'}Z_{12}^{n-m'}Z_{21}^{t'}Z_{22}^{l-t'}\hat{k}^{n+l-(m'+t')}\hat{m}^{m'+t'}\right)C'_{00mk}. \quad \text{(A8)}$$

One should note that, since total momentum of vacuum states should be zero, $C'_{00mk}=0$ unless $m=k=0$. Therefore, in Eq. (A8) only terms with $m'+t'=m, n+l-(m'+t')=k$ must survive and from Eq. (A4) we get $Z_{11}=-Z_{22}, Z_{12}=Z_{21}$ to find

$$|0,\theta\rangle=\mathcal{Z}\sum_{n,l=0}^{\infty}\sum_{m=0}^{n+l}\frac{B_{nlm}}{n!l!}(\hat{a}_1^\dagger)^n(\hat{a}_2^\dagger)^l(\hat{b}_{-1}^\dagger)^m(\hat{b}_{-2}^\dagger)^{n+l-m}|0\rangle, \quad \text{(A9)}$$

where

$$B_{nlm}=\sum_{\substack{m'+t'=m\\0\le m'\le l\\0\le t'\le n}}C_n^{m'}C_l^{t'}Z_{11}^{l+m'-t'}Z_{12}^{n-m'+t'}(-1)^{l-t'}. \quad \text{(A10)}$$

Using a direct expansion, one can also verify that the above expression for the vacuum state is equivalent to

$$|0,\theta\rangle=\mathcal{Z}\sum_{n,l}\frac{1}{n!l!}(Z_{11}a_1^\dagger b_{-1}^\dagger+Z_{12}a_1^\dagger b_{-2}^\dagger)^n$$
$$\times(-Z_{11}a_2^\dagger b_{-2}^\dagger+Z_{12}a_2^\dagger b_{-1}^\dagger)^l|0\rangle. \quad \text{(A11)}$$

APPENDIX B: PERTURBATIVE EXPANSION IN θ FOR THE FLAVOR VACUUM

In this appendix, we try to directly estimate the norm of the flavor vacuum state $G^{-1}(\theta)|0\rangle=\exp(-\theta\hat{S})|0\rangle$ using the perturbative expansion in powers of θ and show that the perturbative calculation of the flavor vacuum state is indeed impossible for large $\gamma_-\theta$. Truncating the series for $G^{-1}(\theta)|0\rangle$ to the N terms, we have the term with the largest number of particles coming from $(\gamma_-/2)(a_1^\dagger b_{-2}^\dagger+a_1^\dagger b_{-2}^\dagger)$ in the $(-\theta\hat{S})^N$. Thus, the truncated series of $G_N^{-1}(\theta)|0\rangle$ can be written as

$$G_N^{-1}(\theta)|0\rangle=X+\frac{1}{N!}\left(-\frac{\gamma_-\theta}{2}\right)^N(a_1^\dagger b_{-2}^\dagger+a_2^\dagger b_{-1}^\dagger)^N|0\rangle$$
$$=X+\frac{1}{N!}\left(-\frac{\gamma_-\theta}{2}\right)^N\sum_{n=0}^{N}C_N^n(a_1^\dagger)^n(b_{-2}^\dagger)^n(a_2^\dagger)^{N-n}$$
$$\times(b_{-2}^\dagger)^{N-n}|0\rangle, \quad \text{(B1)}$$

where X denotes all terms with the total number of particles and antiparticles less than $2N$. For the norm of the above expression we can then write

$$\|G_N^{-1}(\theta)|0\rangle\|^2=\|X\|^2+\left(\frac{\gamma_-\theta}{2}\right)^{2N}\frac{1}{N!^2}\times\sum_{n=0}^{N} n!n!(N-n)!(N-n)!\frac{N!^2}{n!^2(N-n)!^2}=\|X\|^2+(N+1)\left(\frac{\gamma_-\theta}{2}\right)^{2N}. \quad \text{(B2)}$$

Thus when $\gamma_-\theta>2$ the norm of the $|0,\theta\rangle_N=G_N^{-1}(\theta)|0\rangle$ is growing as the number of terms kept in the expansion of the $G(\theta)$ grows and therefore the transformation operator $G_N^{-1}(\theta)$ is not a well defined operator in the mass-eigenstate Fock space.

One may also try to check directly the identity $G(\theta)G^{-1}(\theta)=1$. In this type of approach one defines

$$G(\theta)=\lim_{N\to\infty}G_N(\theta)=\lim_{N\to\infty}\sum_{n=0}^{N}\frac{(\theta\cdot\hat{S})^n}{n!}. \quad \text{(B3)}$$

Then, one shall prove that $\lim_{N\to\infty}\|G_N(\theta)G_N^{-1}(\theta)-\hat{1}\|=0$, i.e., $\lim_{N\to\infty}\|(G_N(\theta)G_N^{-1}(\theta)-\hat{1})|x\rangle\|=0$ for any mass-eigenstate state $|x\rangle$ if $G(\theta)$ is well defined. When multiplying $G_N(\theta)$ and $G_N^{-1}(\theta)$ one typically get all coefficients vanished till the power of N and then have a ''tail'' up to the $\hat{S}^{2N}$ coefficient. If $G(\theta)$ is well defined, this tail is expected to vanish when N is taken to infinity. However, this does not always happen in the perturbative expansion. To demonstrate this one may consider the last term of the ''tail'' given exactly by $\hat{S}^{2N}/(N!N!)$. Recalling that $\hat{S}$ generator contains $(\gamma_-/2)(a_1^\dagger b_{-2}^\dagger+a_2^\dagger b_{-1}^\dagger)$ combination, we can write the state $(G_NG_N^{-1}-1)|0\rangle$ as

$$(G_NG_N^{-1}-1)|0\rangle=Y+\frac{(\theta\gamma_-)^{2N}}{2^{2N}N!^2}\sum_{t=0}^{2N}C_{2N}^t(a_1^\dagger)^t(b_{-2}^\dagger)^t\times(a_2^\dagger)^{2N-t}(b_{-1}^\dagger)^{2N-t}|0\rangle, \quad \text{(B4)}$$

where Y denotes all states with less then $4N$ number of particles and antiparticles. The norm of this state is then given by

$$\|(G_NG_N^{-1}-1)|0\rangle\|^2=\|Y\|^2+\frac{1}{N!^4}\left(\frac{\gamma_-\theta}{2}\right)^{2N}\sum_{t=0}^{2N}(C_{2N}^t)^2t!t!(2N-t)!\times(2N-t)!\rangle(2N+1)\left(\frac{\gamma_-\theta}{2}\right)^{2N}. \quad \text{(B5)}$$

Again when $\gamma_-\theta>2$ the above expression is not convergent and the $G(\theta)G^{-1}(\theta)$ expression is in fact not well defined in terms of mass-eigenstate fields.

For small values of θ, however, the perturbative expansions are indeed convergent and the radius of convergence is related to the pole of $\mathcal{Z}=\langle 0|0,\theta\rangle$ on the complex plane of θ. Here, the pole (critical) value is given by $\gamma_-\sinh(\theta_{critical})=2$.

APPENDIX C: REMARKS ON THE GREEN FUNCTION METHOD AND THE ARBITRARY MASS PARAMETRIZATION

1. Green function method

We note that a straightforward use of the Green function with the conventional definition $\langle 0|T[\psi(x)\overline{\psi}(y)]|0\rangle$ encounters some difficulties in the mixing analysis due to the fact that the flavor vacuum state is not stationary in time $[|0,\theta\rangle(t)\neq|0,\theta\rangle(t')]$. The conventional Green function cannot be adopted without specifying at which times the flavor vacua were taken in the inner product. In fact, the most obvious generalization of the Green function as the overlap between the states created at times x^0 and y^0 [i.e., $G(\alpha\to\beta;x^0,y^0)=\langle 0,y^0|T[\psi_\beta\overline{\psi}_\alpha]|0,x^0\rangle$] breaks down due to the unitary inequivalence of the flavor Fock spaces at different times. Therefore, some sort of modification, such as parallel translation of states to the same time, shall be needed to define the Green function appropriate in the mixing analysis. The flavor mixing problem can then be treated using this modified propagation functions as discussed in the previous literature [10].

In the process of our calculations we also noticed that some entities indeed appeared as ''transition'' amplitudes from one state to another. For example, $\langle N_\alpha\rangle=Z_\alpha+|A_{\alpha\alpha}|^2$ can be considered as superposition of ''vacuum rotation'' background contribution Z_α and contribution from $\alpha\to\alpha$ transition with $A_{\alpha\alpha}$ transition amplitude. In this manner, one can introduce the Green function that only accounts for the transition amplitude without the vacuum contribution. In this way the Green function $(t>0)$ can be defined by

$$G_{\alpha\alpha}(x,t;y,0)=\langle 0'|\varphi_\alpha(x,t)\varphi_\alpha^\dagger(y,0)|0'\rangle, \quad \text{(C1)}$$

where the vacuum state is taken at any (but certain) fixed time, for example $t=0$, and this coincides with the definition given in [10]. We note that at $t=0$ the modified Green function is proportional to the delta function in the position space and thus it vanishes in the spacelike region. Also, the modified Green function satisfies the same field equation that the field operators satisfy even if t is not zero. Thus, to the extent that the field equations satisfy the causality, the modified Green function satisfies the same.[1] For the propagator with a definite momentum $\vec{k}$, we then obtain

$$G_{\alpha\alpha}(\vec{k},t)=A_{\alpha\alpha}^*(\vec{k},t),$$

$$G_{\alpha\beta}(\vec{k},t)=A_{\beta\alpha}^*(\vec{k},t), \text{ etc.} \quad \text{(C2)}$$

[1]See Ref. [18] for the discussion of causality that the Green function should satisfy.

Equation (3.7) then allows to define the propagation functions for any kind of transition.

We should note, however, that treatment with such a modified Green function does not cover all the variety of the effects in the mixing problem. In particular the condensate contribution Z_α, which can be related to the unitary inequivalence of the flavor Fock spaces at different times, is lost so that this part of the problem is missing when the above approach is taken. Nevertheless, the Green function method is useful in calculations of flavor-operator expectation values, scattering amplitudes and other quantities in which the vacuum contribution Z cancels out.

2. Arbitrary mass parametrization

In this sub-appendix, we remark on the arbitrary mass parametrization [11,13] in the mixing problem. As discussed in Ref. [13], one may treat the flavor fields that were initially written as

$$\phi_\alpha = \int \frac{d\vec{k}}{(2\pi)^{3/2}} [u_{k,i} a_{k,i}(t) + v_{-k,i} b^\dagger_{-k,i}(t)] e^{i\vec{k}\vec{x}}, \quad \text{(C3)}$$

equally well in an arbitrary mass basis, i.e.,

$$\phi_\alpha = \int \frac{d\vec{k}}{(2\pi)^{3/2}} [\tilde{u}_{k,i} \tilde{a}_{k,i}(t) + \tilde{v}_{-k,i} \tilde{b}^\dagger_{-k,i}(t)] e^{i\vec{k}\vec{x}}, \quad \text{(C4)}$$

where $\tilde{u}_{k,i}$ and $\tilde{v}_{k,i}$ are free-field amplitudes with some new arbitrary masses. Since there is no physical reason to prefer one form over the other, Ref. [13] and then Ref. [11] claimed that no arbitrary mass parameters should appear in physically observable quantities, i.e., they shall be invariant under specific Bogoliubov transformation going from Eq. (C3) to Eq. (C4) [11]

$$\begin{pmatrix} \tilde{a}_i(t) \\ \tilde{b}^\dagger_i(t) \end{pmatrix} = J^{-1}(t) \begin{pmatrix} a_i(t) \\ b^\dagger_i(t) \end{pmatrix} J(t). \quad \text{(C5)}$$

It is true [11,13] that the perturbative vacuum treatment [8,9] yields the normalization of the flavor state not as unity but as some constant that depends on the arbitrary mass parameter. In particular, Eq. (3.3) can be viewed as an expansion of the flavor ladder operator in some basis constructed from the free-field ladder operators. Then, the normalization of the one-particle state in perturbative vacuum treatment [9] was given only by the $|S_-|^2$ coefficient at the $b^\dagger_{1,2}$ operator, which is obviously dependent on the choice of basis, e.g., changes with rotation in $\{(a_1,a_2)\text{-}(b_1,b_2)\}$ plane. Such arbitrariness is completely avoided in the exact vacuum treatment because the normalization of the flavor state is given by unity no matter what mass basis is used.

However, the claim [11,13] that the number expectation values are not physical because they do depend on the arbitrary mass parameters cannot be correct. As a counter example to such claim, one can consider a very specific case of the mixing problem, namely, when the mixing is absent $[G(\theta)=1]$. As we discuss below, applying such a claim [11,13] to this specific example leads to a conclusion that cannot be correct. With no mixing, we are dealing with nothing else but a free-field problem where the particle number operator $N_i(t)=a^\dagger_i(t)a_i(t)$ and the particle number expectation value $\langle N_i\rangle = |\{a_i(0), a^\dagger_i(t)\}|^2$ (for simplicity we consider the fermion case) must be well-defined physically observable quantities. However, when we apply the transformation given by Eq. (C5) and compare directly Eq. (C3) and Eq. (C4), we observe that

$$\begin{aligned} \tilde{a}_i &= (\tilde{u}^\dagger_{k,i} u_{k,i}) a_{k,i} e^{-i\epsilon_{k,i}t} + (\tilde{u}^\dagger_{k,i} v_{-k,i}) b^\dagger_{-k,i} e^{i\epsilon_{k,i}t} \\ &= \rho_k a_{k,i} e^{-i\epsilon_{k,i}t} + \lambda_k b^\dagger_{-k,i} e^{i\epsilon_{k,i}t} \end{aligned} \quad \text{(C6)}$$

and

$$\langle \tilde{N}_i\rangle = |\{\tilde{a}_i, \tilde{a}^\dagger_i(t)\}|^2 = ||\rho_k|^2 e^{-i\epsilon_{k,i}t} + |\lambda_k|^2 e^{i\epsilon_{k,i}t}|^2, \quad \text{(C7)}$$

where we follow the notations in Refs. [11,13]. In the case of free fields the number expectation value does depend on the arbitrary mass parameters and thus following the above claim one may conclude that $\langle N_i\rangle$ is not a physically measurable quantity. However, this cannot be correct because both the particle number operator and the number expectation value in the free-field problem are well-defined physical observables. We view the above inconsistency as follows. The transformation given by Eq. (C5) is in fact nothing else but a redefinition of the particle states, so that the tilde quantities correspond to some new quasiparticle objects and the number operator now describes the number of a different type of particles than before. Therefore, the number operator average shall not be expected to be the same in such transformations. Indeed, it should change in some covariant and self-consistent manner instead of being invariant under such a redefinition.

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