

Molar mass estimate of dark matter from the dark mass distribution measurements

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We study the distribution of dark matter versus visible matter using a set of data obtained from strong gravitational lensing in the galaxy cluster CL0024+1654 and another set of data inferred from the universal rotation curves in spiral galaxies. The important feature of these two dramatically different observations is that the mass density profile of both visible and dark components can be estimated. From these measurements we deduce the mass of the dark matter particle and our estimate of the mass for the dark matter particle is $\mu_d \approx (200 \sim 800)\text{MeV}$. We contrast our estimates from CL0024+1654 data and the universal rotation curves of the spiral galaxies and discuss their consistency.

I. INTRODUCTION

The problem of dark matter has been one of the biggest puzzles in astrophysics for more than a decade. First revealed in galaxy halos through the so-called rotation curves, it was later found in even larger scales such as in the clusters of galaxies. Nowadays, the dark matter is an important and intrinsic part of the cosmological picture of the universe. Even though nobody doubts the existence of this hidden substance these days, little unless nothing is known about its microscopic properties. Many candidate particles have been suggested for the composition of dark matter from various possible standard model extensions but their existence has not yet been confirmed experimentally.

In this paper, we are going to undertake a rather different approach and attempt to estimate the dark matter parameters from the available experimental data based on the simple but general physical principles. In the next section (Section II), we begin with the mass profile of CL0024 galaxy cluster measured by *Tyson et al.* in 1998 [1] and show a way for these data to reveal an amazing correlation between dark and visible mass distributions. Using a simple thermodynamic point of view, we relate the parameters of this correlation with the molar mass ratio of visible and dark matter. In section III, we extend our thermodynamic approach to the case of spiral galaxies and analyze the Universal Rotation Curves (URC) as our input [2] to see if we can strengthen our conclusions. Remarkably, the estimate for the molar mass ratio in this dramatically different system comes out consistent with the result of section II. In section IV, we present possible interpretations of our findings and deduce that they indicate the mass of dark matter particle between 200MeV and 800MeV. Discussion and conclusion follow in section V. In Appendices A and B, the details of our URC analysis and the scaling properties of CL0024 mass profile are presented, respectively.

II. ANALYSIS OF THE MASS PROFILE OF GALAXY CLUSTER CL0024

The idea that the mass distribution in galaxies or galaxy clusters can be measured from the data on gravitational lensing is not new [3]. In the last decade many systems exhibiting gravitational lensing have been studied [1,4–8]. Among them, the study by *Tyson et al.* in 1998 [1] is distinguished in that the detailed mass map was presented both for total and visible mass components. Assuming that the dark matter follows the known thermodynamical laws, we now note that the comparison of these two maps may give a valuable insight in properties of dark matter particles from the experimental perspective.

A brief survey of the measurement is in place. In 1998 the study of Hubble telescope images of galaxy cluster CL0024+1654 has been carried out by *Tyson et al.* and the mass density profile of the cluster has been derived from strong gravitational lensing [1]. It was found that the vast majority of the mass is not associated with the galaxies and forms a smooth elliptical distribution, slightly shallower than isothermal sphere, with a soft core of $r_{core} = 35 \pm 3h^{-1}kpc$, where h is the normalized Hubble constant. No evidence of in-falling massive clumps has been found for the dark component. The projected dark matter density profile is well fit by a power law model

$$\Sigma(y) = \frac{K(1 + \eta y^2)}{(1 + y^2)^{2-\eta}}, \quad (1)$$

where $y = r/r_{core}$, $K = 7900 \pm 100hM_{\odot}pc^{-2}$, $r_{core} = 35 \pm 3h^{-1}kpc$ and $\eta = 0.57 \pm 0.02$ [1,9]. The primary conclusion from the observed mass distribution was that the predicted profile in Ref. [10] was inconsistent with the observed result.

In this section, we attempt to study the mass density profile in CL0024 from a thermodynamic point of view. In particular, we assume that the dark matter complies with the known thermodynamic laws and that it is in the classical region so that Boltzmann statistics can be applied. If the rotation can be neglected, the general thermodynamics principles imply the following distribution for visible and dark mass components in a gas cloud [11]

$$\rho_i(r) = a_i e^{-\beta_i(r)\mu_i\Phi(r)}, \quad (2)$$

where μ_i is the molar mass for the corresponding component, $\beta_i = 1/kT_i$ and $\Phi(r)$ is the gravitational potential at position r . Even though the details of the radial distribution of the gravitating gas may be difficult to model and understand [12,13] (see *e.g.* Appendix B for some discussion on the radial mass distribution in CL0024), Eq.(2) leads to a simple relation between mass densities of dark and visible components, *i.e.*

$$\ln \rho_i(r) - d_i = -\mu_i\beta_i(r)\Phi(r), \quad (3)$$

where $d_i = \ln a_i$. One may expect therefore to see a linear correlation between $\rho_v(r)$ and $\rho_d(r)$ by plotting the data on Log-Log scale. The slope κ is then given by the ratio of the Boltzmann factors for visible and dark matter, *i.e.* ($\kappa \approx \beta_v\mu_v$)/($\beta_d\mu_d$). Although the local fluctuations in the mass density of one component might locally shift $\ln(\rho_i(r))$, outside of the fluctuation region we would still expect to see a line with a certain slope. This motivates us to study these distributions in CL20024 from the assumption that elementary thermodynamics laws can be applied to the analysis of the mass density profile including the dark matter halos. Note that the "linear regime" in Log-Log plot of visible vs. dark matter distribution can be easily spoiled by many factors, *e.g.* non-equilibrium processes, local fluctuations or effects of rotation. Thus, the validity of this thermodynamic assumption must be carefully evaluated from the consistency of the results obtained from the analysis of different systems, as we discuss in the present and next sections (Section II and III).

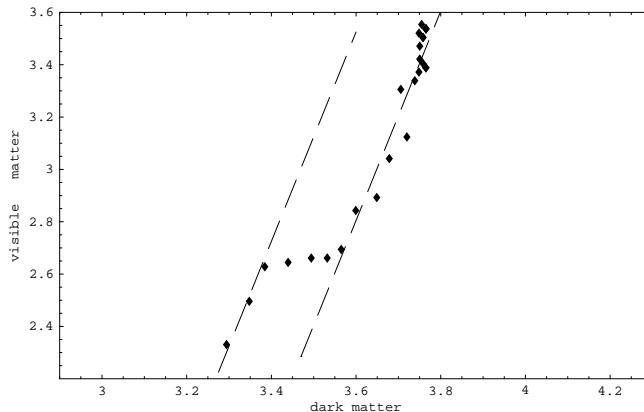


FIG. 1. Data set extracted from mass profiles of CL0024 [1].

In Fig.1, we present the sample of points taken from the graph for the radial projected mass distribution of dark and visible matter by Tyson *et al.* [1] in the Log-Log scale. In the plot we indeed observe a linear correlation. The short horizontal segment on the graph, once related to the radial mass distributions from Tyson *et al.*, could indicate presence of two major subsystems in the visible component of the cluster: inner with smaller spread, and outer with larger spread. We can attribute the inner subsystem to the region of gravitational influence of the central formation in the cluster, while the outer subsystem corresponds to the concentrations of matter formed outside of this region. Transition between the two subsystems occurs in relatively narrow region at distance of about $100 h^{-1}$ kpc, as can be observed from the original plots by Tyson *et al.*, and manifests itself as the above mentioned flat segment. While this effect is rather interesting and deserves further attention in its own, we will leave it aside for the sake of main point in the present discussion.

Outside of the transition region, remarkably, the datum reveals linear correlations with the same slopes. To extract this slope we may further deal with the dataset by either cutting off all points beyond the bridge or by cutting out only the transition region and keeping the other points in the set:

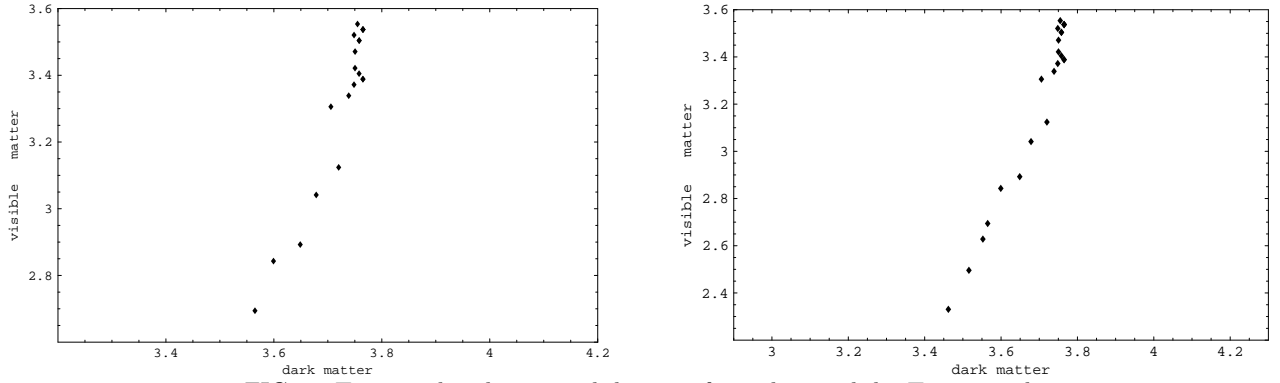


FIG. 2. Truncated and corrected datasets from the graph by Tyson *et al.*

In both cases, indeed, the estimate for the slope of the correlation is similar. For the truncated set one obtains $\kappa \approx (3.55 - 4.9)$ while for the corrected set one obtains $\kappa \approx (3.61 - 4.35)$. Guided by the above thermodynamical idea, we can conjecture from these plots that the ratio of the Boltzmann factors for dark mass distribution and visible mass distribution is consistent with a constant given by $\kappa \approx (3.6 - 4.4)$.

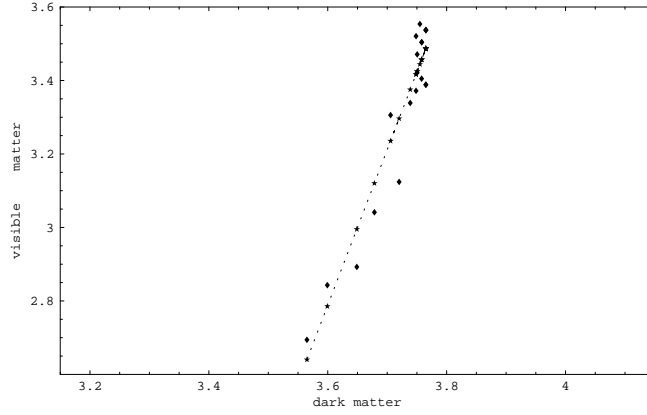


FIG. 3. Linear fit to the data presented in Fig.2.

III. ANALYSIS OF THE UNIVERSAL ROTATION CURVES FOR SPIRAL GALAXIES

It is well known that the rotation curves (RC) of spiral galaxies do not show any Keplerian fall-off beyond their optical radius (see Fig.4). This is one of the most remarkable manifestations of large dark mass component in galaxies. Thus, RCs are indeed the measurements of dark mass distribution in galaxies and it would be interesting to see if the approach presented in Section II yields a meaningful result in this case.

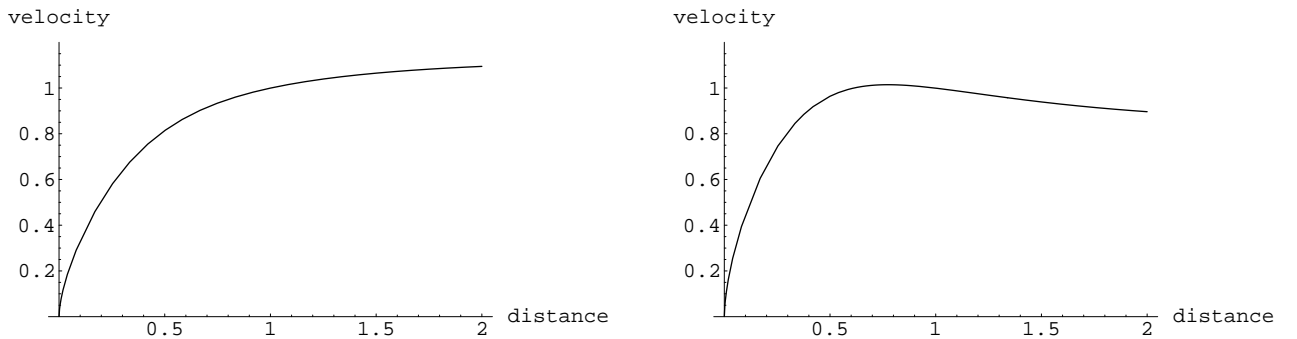


FIG. 4. URC for low luminosity galaxy ($M=-18.5$, left) and high luminosity galaxy ($M=-23.2$, right). Velocity is normalized to v_{opt} , distance is normalized to r_{opt} . See the text for the notations v_{opt} and r_{opt} .

We shall also note that, although the measurement of the mass distribution in CL0024 cluster is a remarkable result, it is quite unique in the sense that there are just few other studies that yield detailed distribution of dark vs. visible matter in galaxy clusters. On the other hand, RCs have been studied for a long time in many galaxies (over 1000 by now) and thus can provide a good consistency check for our method. Moreover, the spiral galaxies and the galaxy clusters are tremendously different systems in their scales and dynamics. The consistency of our approach in these two dramatically different regimes would provide a good evidence of its validity.

Application of the above ideas to the mass distribution in spiral galaxies, however, faces many experimental as well as theoretical complications. It is experimentally difficult and controversial to measure RC and map out the visible mass contribution to it [14,15]. Although a large number of such separations have been carried out [2], the quality of these results is uncertain. Furthermore, the measured RCs rarely extend to more than 1.5-2 optical radius. Thus, in most cases, only a very limited part of dark matter distribution becomes visible that doesn't include the fall-off tail which is the most sensitive part for the properties of dark matter. Also, theoretically, the spiral galaxy is a complex conglomerate of dark matter, interstellar gas, stars and radiation in the presence of essential gravitational pull and rotation. Thus, it is unclear if such a system should satisfy the thermodynamic Boltzmann distribution at all. Even if it does, how the effect of rotation should be included into the analysis is not absolutely clear. Finally, the global temperature variations along the stellar disk may pose a serious problem for the analysis.

In our study, we therefore should check first the consistency of the Boltzmann distribution in spiral galaxies. To begin our study, we accept "the exponential rotating disk plus spherical dark halo" as a conventional mass model for the spiral galaxy and the Universal Rotation Curves (URC) [16] as the experimental input for RCs (see Appendix A).

Note that URC maps out the gravitational potential throughout the galaxy disk via a simple relationship

$$v_{URC}^2(r)/r = \hat{r} \cdot \vec{\nabla} \Phi(r) = \frac{d\Phi(r)}{dr}, \quad (4)$$

so that for the Boltzmann distribution of mass we can write

$$\begin{aligned} \rho_v(r) &= a_v e^{-\beta_v \mu_v E(r)}, \\ E(r) &= \Phi(r) + v_{URC}^2(r)/2, \Phi(r) = \int_0^r dr' v_{URC}^2(r')/r', \end{aligned} \quad (5)$$

where $\beta_v = 1/kT_v$, μ_v is the molar mass of the matter and $E(r)$ is the specific energy (by mass) at distance r including the contribution from the rotation which we assume in the form $v_{URC}^2(r)/2$. Note that, owing to the spherical form of the dark halo, we may assume that the rotation of the dark halo is negligible, *i.e.*

$$\rho_d(r) = a_d e^{-\beta_d \mu_d \Phi(r)}. \quad (6)$$

In Fig.5, we present the radial distribution of the gravitational potential and rotation energy, normalized to $v_{opt}^2 = v_{URC}^2(r_{opt})$, vs. the distance, normalized to the optical radius denoted by r_{opt} . From this figure, one can note that the relative effect of rotation decreases with distance and reduces to almost constant contribution at $r \gtrsim 0.6r_{opt}$. The effect of rotation on the Boltzmann distribution is therefore negligible at these distances.

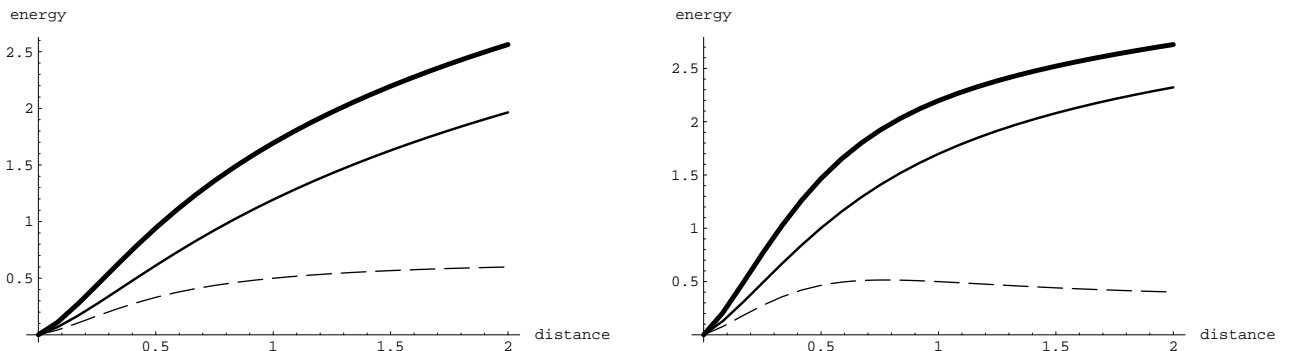


FIG. 5. Total (thick line), gravitational potential (solid line) and kinetic energy (dashed line) in a spiral galaxy mapped out from URC. The left and right plots are for $M=-18.5$ and $M=-23.2$ luminosities, respectively.

To check the consistency of the Boltzmann distribution, we should compare consequently $\ln(\rho_v)$ with $E(r) = \int_0^r dr' v_{URC}^2(r')/r' + v_{URC}^2(r)/2$ and $\ln(\rho_d)$ with $\Phi(r) = \int_0^r dr' v_{URC}^2(r')/r'$, respectively. Despite many possible complications mentioned above, for the distribution of visible mass, we do find a good linear correlation between $\ln(\rho_v)$ and

$E(r)$ for the entire range of available URCs as shown in Fig.6. The correlation gets worse for galaxies with higher luminosities which we may attribute to increasing global variations of temperature along the stellar disk and increased effects from the galaxy bulge.

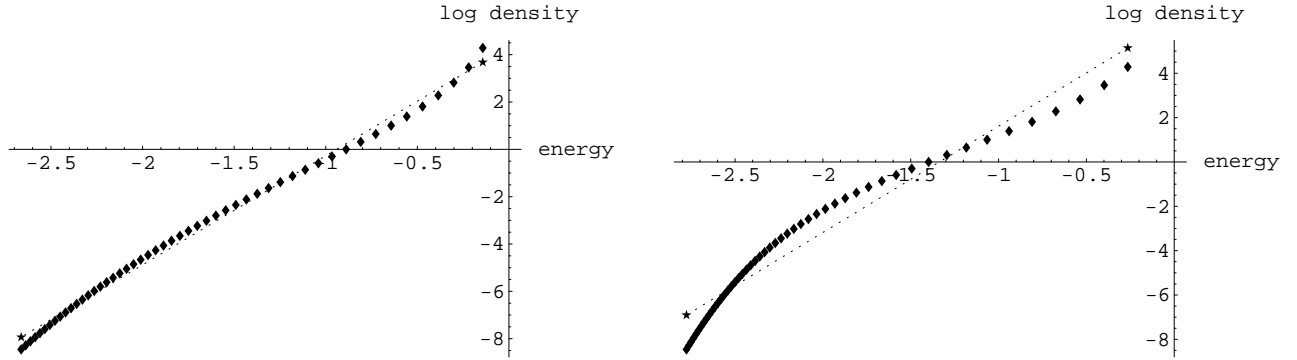


FIG. 6. Visible mass distribution vs. energy for $M=-18.5$ (left) and $M=-23.2$ (right).

For the dark mass, as shown in Fig.7, we observe much worse situation since the linear regime is reached only in the outer part of the stellar disk $r \gtrsim 0.6r_{opt}$.

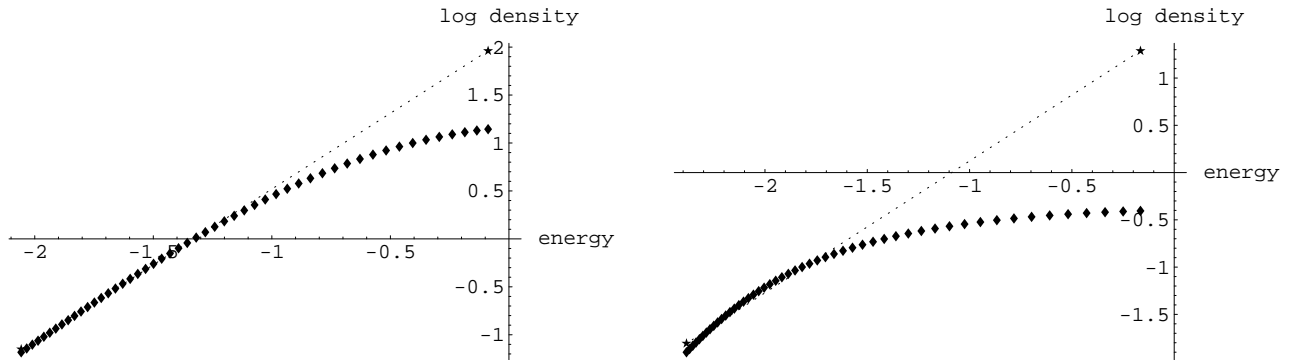


FIG. 7. Dark mass distribution vs. energy for $M=-18.5$ (left) and $M=-23.2$ (right).

While this might be used to argue against our assumptions, we must also note that the dark matter distribution is usually inferred from the separation of RC and thus is subject to a large uncertainty. This is quite different from the case of visible mass distribution which is modeled by the light/mass ratio from the photometric data. Indeed, in Fig.8, we compare the two quite different dark mass distributions, used in [16] and [2], *i.e.* solid and dashed lines, respectively. Both give perfect URC fits but they behave quite differently in small r region.

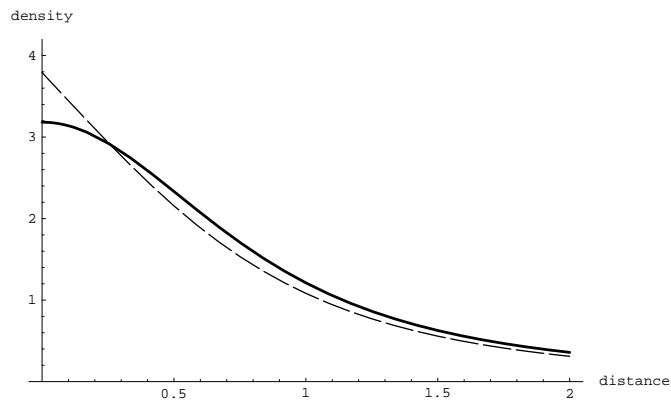


FIG. 8. Two sample density profiles that give perfect fits to URC.

We must therefore conclude that the URC is not sensitive to the details of small r behavior of the dark mass distribution and the discrepancies in Fig.7 cannot be used to ultimately discard the validity of Boltzmann distribution in dark halos. Instead, we should check if an acceptable fit to URC can be made by assuming the dark mass distribution in the form given by Eq.(6).

We performed such fits for a wide range of luminosities from $M=-18.5$ to $M=-23.2$ and found that a perfect fit can be constructed by varying a_d and $\kappa_d = \beta_d \mu_d$ in every case that we considered.

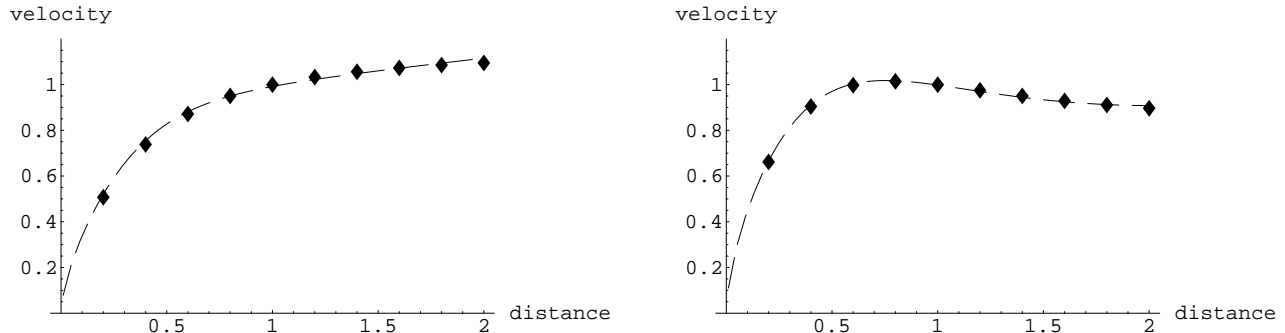


FIG. 9. Boltzmann distribution fit to URC for $M=-18.5$ (left) and $M=-23.2$ (right).

We also found that, although the best fit κ_d for each luminosity M varies with M ($\kappa_d \approx 0.8 - 1.2$), the URC is not very sensitive to κ_d and very good fits can be obtained in all cases even if κ_d is kept constant and *only* a_d is varied.

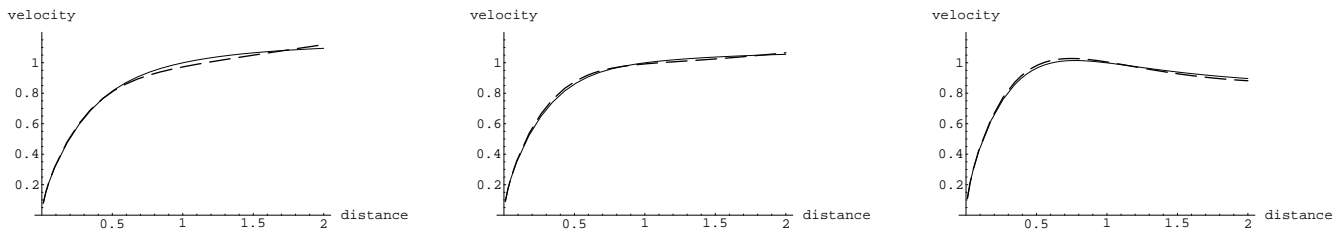


FIG. 10. Boltzmann distribution fit (dashed line) with $\kappa_d \approx 1.1$ kept constant for low ($M=-18.5$, left), intermediate ($M=-20$, center) and high ($M=-23.2$, right) luminosity galaxy.

From the above discussion, we can conclude therefore that the global profile of visible and dark mass in spiral galaxies is consistent with almost isothermal Boltzmann distribution. Even though the data of visible and dark mass distribution plotted in Figs.6 and 7 only partially justify the use of the linear regime, we have found that it is satisfactory in explaining the URCs plotted in Figs.9 and 10. The temperature of the distribution can be estimated as

$$T \sim \frac{\mu_{H_2} v_{opt}^2}{\kappa_v k_B} \sim 10^5 - 10^6 \text{K}, \quad (7)$$

which is consistent with the temperature of the interstellar gas.

Consequently, we found that the ratio $\kappa_v/\kappa_d \approx 5$ for a range of galaxy luminosities. Since we used a synthetic URC as our input data and a synthetic mass model for the visible mass distribution, it is impossible to properly estimate the error in κ and we assume the variance of κ with M in our URC-fits, e.g. $\kappa_d \approx 0.8 - 1.2$, as such an estimate. In this case $\kappa_v \approx 4.7 \pm 0.1$ and $\kappa_d \approx 1.0 \pm 0.2$, from which we obtain the ratio

$$\kappa = (3.7 - 5.7). \quad (8)$$

Our result here is consistent with analysis of the CL0024 galaxy cluster. This is quite remarkable since there is a tremendous difference in the systems under consideration. A lot of complexities in dynamics of spiral galaxies might have affected our thermodynamic picture. Nevertheless, in both situations we may conclude that the ratio of the Boltzmann factors for visible and dark matter is a constant. Including both of our analysis presented in Sections II and III, the constant can be estimated as $\kappa = 3.6 \sim 5.1$.

IV. INTERPRETATION OF THE RESULT

As was already mentioned in Section II, κ can be related to the ratio of molar masses for the visible and dark components $\kappa = (\beta_v/\beta_d)(\mu_v/\mu_d)$. In the previous sections (Sections II and III), we found that this ratio is constant in a variety of very different conditions such as the galaxy clusters and the spiral galaxies of different luminosities. Although it is still an open question how the two apparently different systems, *i.e.* dark matter and visible matter, could share the same temperature, it is essentially the local thermodynamic equilibrium which we think is the most natural way to consistently describe the data with such variety of different conditions. Since the temperature (β) and the molar mass (μ) are independent variables, it is unlikely that μ_v/μ_d varies in a precise correlation with β_v/β_d in such a way that κ is a constant both for the CL0024 galaxy cluster and the spiral galaxies with various luminosities. This leads rather naturally that β_v/β_d must be a constant within about 20% from our analyses presented in Sections II and III. Moreover, the temperature is likely to vary with the distance as shown in Appendix B but yet no significant change in κ is noticeable in the mass profiles of CL0024. Also, the temperature given by Eq.(7) vary widely even though κ_v and κ_d vary insignificantly as shown in Section III, since it is proportional to v_{opt}^2 , yet the ratio β_v/β_d is consistent with constant. Due to such correlation between β_v and β_d evident in our analysis, we think that the local thermodynamic equilibrium between the dark and visible matter, *i.e.* $\beta_d(r) \approx \beta_v(r)$, is the most natural way to describe these data. This might imply that the gravitational scattering in the dark matter may create non-vanishing relaxation effect at the scales and times of the galactic halos. Feasible mechanism to reach the local thermal equilibrium along this line may deserve further investigations. Under the local thermodynamic equilibrium between dark and visible matter

$$\kappa \approx \mu_v/\mu_d. \quad (9)$$

Hence, our result gives us a possibility to estimate μ_d once μ_v is known. Since the ordinary matter primarily consists of H , H_2 and He [17], we immediately conclude that the range for the mass of dark matter particle is from 150MeV to 1250MeV. The exact number, however, may depend significantly on the exact composition of visible matter.

Indeed, if one takes into account that the visible matter is made of few components which are in fairly strong gravitational field, one finds that the relative content of matter in such conditions changes dramatically with the distance (*e.g.* In Fig.11, the upper and lower lines represent the densities of H_2 and He , respectively, as functions of the logarithmic distance; they are normalized to 1 at the origin) [18].

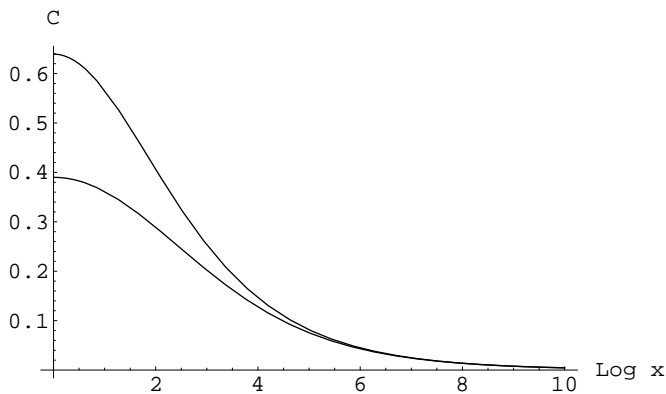


FIG. 11. Relative content of two-component gas in strong gravitational field.

One may ask then if the linear correlation is at all to be expected between $\ln(\rho_d)$ and $\ln(\rho_v) = \ln(\rho_{H_2} + \rho_{He})$. To answer this question we perform a simulation for such situation with different relative contents for visible mass.

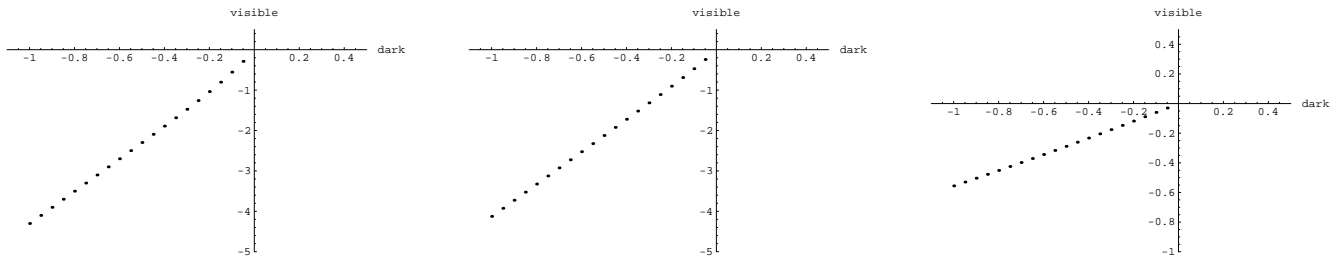


FIG. 12. $\ln \rho_v$ vs $\ln \rho_d$ when the dark matter particle is light (500MeV)(see the left two plots) and heavy (4GeV)(see the right plot). For the left two plots, the visible component has the composition of 75% H_2 + 25% He (left) and 50% H_2 + 50% He (center), respectively. For the right plot, the visible component has the composition 75% H_2 + 25% He.

When this effect is taken into account, the correlation is still linear although the different relative contents obviously affect the slope of the line a little. We conclude therefore that when the composite nature of the interstellar medium is taken into account the linear correlation between $\ln(\rho_v)$ and $\ln(\rho_d)$ is still preserved. The slope of this correlation is not very sensitive to the exact concentrations although the knowledge of the visible mass composition is needed to reliably extract the mass of dark matter particle from the slope of the correlation. From this and the above results we can estimate the mass of dark matter particle, including the uncertainty in visible mass content, as

$$\mu_d \approx (200 \sim 800)MeV = 500 \pm 300MeV.$$

In more details, our error estimates include other uncertainties. The statistical uncertainty in κ we find to be $\epsilon_1 \approx 20\%$. The uncertainty due to the effects of rotation can be estimated by considering the case of spiral galaxies with and without correction due to rotation in which case we find variance of about $\epsilon_2 \approx 20\%$. As we discussed in the beginning of this section, we assumed the local thermodynamic equilibrium between dark and visible matter, *i.e.* $\beta_v(r)/\beta_d(r) \approx 1$. Although in the absence of any direct measurements of the properties of the dark matter it is simply impossible to estimate the uncertainty on $\beta_v(r)/\beta_d(r)$, we may hope that the variation due to this ratio in part revealed itself through variation in measurements of κ and thus may not be larger than the statistical uncertainty in κ . Taking these uncertainties into account for given κ we may relax the range to $\mu_v/\mu_d \approx 3.0 \sim 6.0$. The uncertainty in the visible matter composition is more difficult to estimate due to absence of reliable measurements as well as due to the fact that this composition may vary greatly throughout the space. We estimate this uncertainty as due to the uncertainty in content of atomic and molecular H in which case $\mu_v \approx (1200 \sim 2100)MeV$. Accumulating these uncertainties we get $\mu_d \approx (200 \sim 800)MeV$.

V. DISCUSSION AND CONCLUSION

We studied the mass distribution in the CL0024 galaxy cluster and RCs for the spiral galaxies from the thermodynamic point of view. We showed that the global visible and dark mass distribution in spiral galaxies is consistent with an almost isothermal Boltzmann distribution. We find that the μ/T factors of these distributions are close to a constant for a wide range of galaxy luminosities both for visible and dark matter. Consequently, we find that the ratio of μ/T 's for visible and dark matter is a universal constant of approximately $\kappa \approx 4.4 \pm 0.8$. The same conclusion is also drawn from the analysis of mass distribution in the galaxy cluster CL0024.

The simplest interpretation of this result implies that the visible matter is approximately 4 times heavier than the dark matter. If we narrow down the composition of visible matter as the one in the interstellar medium suggested in Ref. [17], *i.e.* 45% H, 45% H_2 and 10% He, then we get that the dark matter is made of particles with mass of about 300MeV. To our knowledge, there are no candidates of mass in this range within the current extensions of Standard Model. Massive neutrinos and axions are usually ruled out by the cosmological arguments of large structure formation. Also, their experimental mass limits $< 25MeV$ for the heaviest τ -neutrino [19] are well below our range. The most favorable candidates for non-baryonic dark matter, such as Weakly Interacting Massive Particle (WIMP) with the mass anywhere between 10GeV and 1TeV and the SUSY lightest particle, *e.g.* neutralino, with the mass above 30GeV [20], are also far out of the range yielded from our analysis. In fact, any existing dark matter particle candidate would result in either too stretched (neutrinos and axions) or too shrunk (WIMP & neutralino) halos and thus have difficulties explaining observed extent of dark halos. It is however interesting to note that the energy scale of 300MeV is astonishingly close to the QCD scale Λ_{QCD} and constituent u-d quark masses. Whether our finding is indeed related to the QCD vacuum properties may deserve a further study.

Because our analysis is based on the assumptions of the local statistical equilibrium of dark and visible matter and the equal temperature between the visible and dark matter components, we add some more comments here' risk of repeating ourselves. The hypothesis of linear regime in Log-Log plots of dark vs. visible matter distributions is motivated by very elementary thermodynamics reasoning. This regime, however, can be easily spoiled by various non-equilibrium and rotation effects. Thus the validity of this hypothesis can be verified only from the consistency of the final results. In our study we do find consistent estimates for κ from two tremendously different classes of systems and we do find that the hypothesis of linear regime is satisfactory in explaining the dark mass distribution in galaxy cluster CL0024 as well as the URC in spiral galaxies of different luminosities. Furthermore we find that the ratio of factors μ/T for the visible and the dark components is constant up to about 20%. While it is still possible that the independant variances in the temperature of visible and dark matter are all within our measurement uncertainty,

this implies that β_v and β_d vary in a correlated way so that the ratio β_v/β_d is preserved as a constant. Under these conditions the most feasible explanation is that the ratio β_v/β_d is unity due to thermodynamic equilibrium which implies the local statistical equilibrium and the equal temperature between the visible and dark matter components. Unfortunately we do lack crucial experimental evidences to provide the decisive argument here or to estimate properly uncertainty caused by this assumption. Yet, if the assumption of local thermal equilibrium is to be discarded, one needs to suggest other physical mechanism that could provide constant observed dark to visible temperature ratio other than 1.

Also it may be still possible that μ in global Boltzmann distributions might be something other than the molar mass of the matter. Yet, the molar mass seems to be the most natural choice for μ and the consistency of the temperature of the visible matter distribution in this case shows that this choice is, indeed, the most viable variant. Finally, we also neglected the effect of rotation for the dark component (and in the case of the galaxy cluster CL0024 we also neglected the rotation effect of visible component) that could alter the value of κ . Such assumption, however, was natural for the given smooth elliptical form of dark halos. Still we estimate the uncertainty due to rotation from considering spiral galaxies with and without effects of rotation and include it in the total uncertainty of our final estimate for μ_d .

To summarize, in our study, we found that the isothermal Boltzmann distribution is satisfactory in explaining observed visible and dark matter profiles in spiral galaxies. The factors of these distributions are constant over a wide range of galaxy luminosities. The ratio of these factors for visible and dark matter is a constant. If this constant is related to the ratio of the molar mass of dark matter to that of the visible component, then it gives an estimate for the mass of dark matter particle: $\mu_d \approx (200 \sim 800)$ MeV. The measurement in the galaxy cluster CL0024 along with the fact that the lightest component of the visible matter most significantly affects κ , as was pointed out in Section IV, indicates that the mass of the dark matter particle is in fact closer to 300MeV, which appears astonishingly close to QCD energy scale.

APPENDIX A: THE UNIVERSAL ROTATION CURVES OF SPIRAL GALAXIES

The study in Ref. [2], involving the samples of more than 1000 spiral RCs, has shown that RCs grouped by luminosity follow certain universal profiles. These profiles were called Universal Rotation Curves (URC) and it was shown that they can be well fitted for a wide range of luminosities with a simple mass model including exponential disk and spherical halo.

According to the synthetic luminous mass profiles, the mass distribution in stellar disks can be well fitted by assuming $\rho_v \sim r^{-2} \exp(-3.2r/r_{opt})$. The self-gravity on an infinitely thin disk with the surface mass density $I(x) \sim \exp(-3.2x)$, ($x = r/r_{opt}$) yields an equilibrium circular velocity given by

$$v_v^2(x) = 1.28\beta v_{opt}^2 x^2 (I_0(1.6x)K_0(1.6x) - I_1(1.6x)K_1(1.6x)), \quad (\text{A1})$$

where $\beta = v_v^2(r_{opt})/v_{opt}^2 = 1.1GM_v/v_{opt}^2 r_{opt}$ and I_0, I_1, K_0, K_1 are the Bessel functions. In [2], this contribution is fitted with a simpler formula valid in the range $x \lesssim 2$:

$$v_v^2(x) = v_{opt}^2 \beta 1.97 \frac{x^{1.22}}{(x^2 + 0.782)^{1.43}} \quad (\text{A2})$$

which we used in our calculations. Beyond $2r_{opt}$, the Keplerian regime is practically attained so that $v^2(r)_v = v_v^2(2r_{opt})(2r_{opt}/r)$.

The contribution from a dark halo can be well represented by

$$v_d^2(x) = v_{opt}^2 (1 - \beta) (1 + \alpha^2) \frac{x^2}{x^2 + \alpha^2} \quad (\text{A3})$$

with α being the ‘velocity core radius’ normalized to r_{opt} . The URC profile then can be very well fitted by assuming

$$\begin{aligned} v_{RC}^2(r) &= v_d^2(r) + v_v^2(r), \\ \beta &= 0.72 + 0.44 \log(L/L_*), \\ \alpha &= 1.5(L/L_*)^{1/5}, \end{aligned} \quad (\text{A4})$$

with L being the I-band luminosity $L/L_* = 10^{-(M+21.9)/5}$. Eq.(A4) reproduces the URC for various luminosities M in the range from -18.5 to -23.2 within their rms. Small r behavior of the URC is dictated by stellar disk and is close to $v^2 \sim r$, while at large distances URC is dominated by dark halo contribution (especially for smaller galaxies) and is close to constant.

APPENDIX B: INTERESTING SCALING PROPERTIES OF CL0024 MASS PROFILE

As was pointed out the observed mass density for dark matter is typically inconsistent with the simulations with the cold dark matter (CDM) model. Another naive try would be to consider an isothermal gas cloud model. If one neglects the rotation, fairly simple result for the mass density profile can be obtained. This case is known as the Lane-Emden equation [21,22]

$$\frac{d^2\omega}{dy^2} + \frac{2}{y} \frac{d\omega}{dy} = e^{-\omega}, \quad (\text{B1})$$

where y is the distance normalized to $r_0 = \sqrt{\sigma^2/4\pi G\rho_0}$ (ρ_0 is the mass density at the origin, *i.e.* $r = 0$) and σ is the constant velocity dispersion (related to the temperature of the sphere T and the particle mass μ by $\sigma^2 = kT/\mu$), so that the mass density is given by

$$\rho(r) = \rho_0 e^{-\omega(r/r_0)}. \quad (\text{B2})$$

The isothermal sphere solution has a soft core which is qualitatively consistent with the observed mass distribution in CL0024 [9], however, as more detailed study shows, it falls off too rapidly and as such is not quantitatively acceptable.

If one agrees, however, that isothermal sphere is good in describing some global features of the CL0024 profile, one might expect then to see some of major features of the isothermal solution in experimental profile. One of such features is the specific form of the solution, *i.e.* $\ln(\rho_i(r)/\rho_i(0)) \sim \omega_i(r/r_{0,i})$. On the Log-Log plot this would mean that $\ln(\rho_i/\rho_i(0)) \sim f(z - z_i)$ where $z = \log(r)$. Guided by this idea we compare the experimental mass profiles for the visible and dark components as well as for the total mass. One can see that, indeed, the mass profiles are aligned pretty well with simple horizontal shift. We find that the visible component profile is shifted on the Log-Log plot with respect to the dark component profile by $\Delta z \approx -0.62$, while the total mass profile is shifted by $\Delta z \approx 0.22$. The visible component mass profile is shifted with respect to the total mass profile by $\Delta z \approx -0.4$.

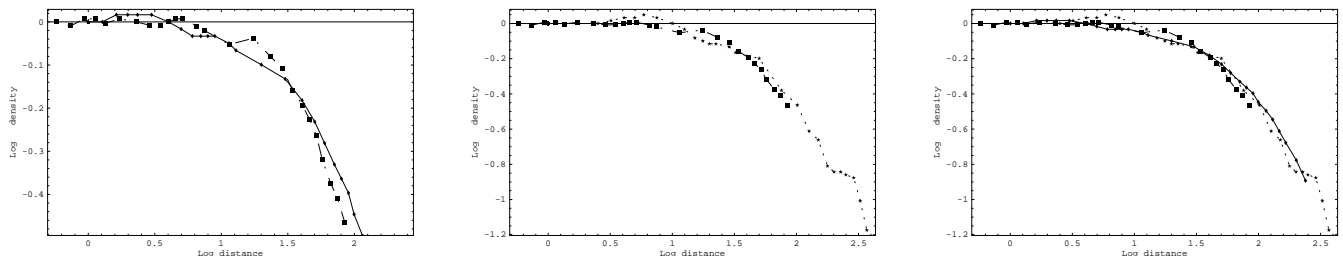


FIG. 13. Alignment by horizontal shift of the total mass (left) and visible mass profiles (center) with the dark mass profile. Alignment of all three mass profiles is presented in the right panel.

We believe this observed similarity can be understood in terms of the dimensionless variables of possible configurations of the rotating (and possibly not isothermal) self-gravitating gas.

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- [1] J.A. Tyson *et al.*, astro-ph/9801193.
 - [2] M.Persic, P.Salucci, F.Stel:MNRAS, 281, 27 (1996).
 - [3] N. Straumann, astro-ph/0108255.
 - [4] X-P. Wu, T. Chiueh, L-Z. Fang and Y-J. Xue, asto-ph/9808179.
 - [5] J-P. Kneib, astro-ph/0009385.
 - [6] H. M. AbdelSalam, P.Saha, astro-ph/9806244.
 - [7] J.Bezecourt & all, astro-ph/0001513.
 - [8] M. White, L. Hernquist, V. Springel, astro-ph/0107023.
 - [9] P.R. Shapiro I.T.Iliev astro-ph/0006353.
 - [10] J.Navaro, C.Frenk, S.White; ApJ, 462, 563 (1996).
 - [11] G.N.Hatsopoulos, J.H.Kennan, *Principles of general thermodynamics*, Wiley, pp.513-515, 1965.
 - [12] E.V. Votyakov & all, Phys. Rev. Lett. **89**, 031101 (2002); cond-mat/0202140.
 - [13] H.J. de Vega, N.Sanchez, Nucl. Phys. **B625**,460-494 (2002); astro-ph/0101567.

- [14] F.Combes, astro-ph/0206126.
- [15] T.S. van Aldaba, J.S.Bahcall, K.Begeman, R.Sancisi:ApJ, 295, 305 (1985).
- [16] P.Salucci, A.Borriello, astro-ph/0203457.
- [17] *The Interstellar Medium in Galaxies*,ed. H.A.Thronson and J.M.Shuller, 1990, Proceedings of 2nd Tetons Conference, Kluwer Publisher, 556 pages.
- [18] H.J. de Vega, J.A. Siebert, Phys. Rev., /bf E66,016112 (2002); astro-ph/0111551.
- [19] W.Hu, D.Eisenstein, M.Tegmark, Phys. Rev. Lett. **80**, 5255-5258 (1998); astro-ph/9712057.
- [20] S.Khalil, C.Munoz, Contemp. Phys. **43**, 51-62 (2002); astro-ph/0110122.
- [21] P. Natarajan, D. Lynden-Bell astro-ph/9604084.
- [22] R. Emden, 1907 in *Gaskugeln-Anwendungen der Mechan. Warm theorie*. Druck und verlag Von B.G. Teubner, Leipzig, p.129.