Phenomenology of flavor oscillations with non-perturbative effects from quantum field theory

Antonio Capolupo[a*] Chueng-Ryong Ji[b†], Yuriy Mishchenko[b‡], and Giuseppe Vitiello[a§]
[a] Dipartimento di Fisica "E.R. Caianiello" and INFN, Università di Salerno, I-84100 Salerno, Italy
[b] Department of Physics, North Carolina State University, Raleigh, NC 27695-8202, USA

We analyze phenomenological aspects of the quantum field theoretical formulation of meson mixing and obtain the exact oscillation formula in the presence of the decay. This formula is different from quantum mechanical formula by additional high-frequency oscillation terms. In the infinite volume limit, the space of the flavor quantum states is unitarily inequivalent to the space of energy eigenstates.

Quantum mixing of particles is among the most interesting and important topics in Particle Physics[1]. The Standard Model involves quantum mixing in the form of Kobayashi-Maskawa (CKM) mixing matrix [2], a generalization of the original Cabibbo mixing between d and s quarks [3]. Also, recently, convincing evidences of neutrino mixing have been provided by Super-Kamiokande and SNO experiments [4, 5, 6, 7, 8], thus suggesting neutrino oscillations as the most likely resolution for the solar neutrino puzzle [9] and the neutrino masses[10]. Since the middle of the century, when the quantum mixing was first observed in meson systems, this phenomenon has played a significant role in the phenomenology of particle physics. Back in 1960s the mixing of K^0 and $\bar{K}^0$ provided an evidence of CP-violation in weak interactions[11] and more recently the $B^0\bar{B}^0$ mixing is used immensely to experimentally determine the precise profile of CKM unitarity triangle[2, 3, 12]. Upgraded high-precision mixing experiments in the meson sector would be vital to search for any deviation from the unitarity of CKM matrix and thus put important constraints on the new physics beyond the Standard Model. At the same time, in the fermion sector, the discovery of neutrino mixing and neutrino masses challenged our fundamental understanding of CP-violation and, therefore, of the Standard Model itself.

Regarding the vanishing magnitudes of the expected new physics effects (such as the

*E-mail: capolupo@sa.infn.it
†E-mail: ji@ncsu.edu
‡E-mail: ymishch@unity.ncsu.edu
§E-mail: vitiello@sa.infn.it

unitarity violation in CKM matrix and/or neutrino masses), it is imperative that the theoretical aspects of the quantum mixing are precisely understood. In this direction, it was noticed recently that the conventional treatment of flavor mixing, where the flavor states are defined in the Fock space of the energy-eigenstates, suffers from the problem of total probability non-conservation [13]. This demonstrated that the mixed states should be treated rather independently from the energy-eigenstates. In fact, it was shown that the Fock space of the mixed states is unitary inequivalent to the Fock space of the energy-eigenstates and that the additional high-frequency term must be present in the flavor oscillation formulas. Simpler quantum mechanical result is reproduced only in the relativistic limit of quantum field theory. In this respect, one may question the magnitude of the field-theoretical effects and their significance to the new physics in the mixing phenomena.

A significant research effort had been undertaken in the quantum field theory of mixing [13, 14, 15, 16, 17, 18, 19, 20, 21]. Still, the general theoretical results obtained therein cannot be immediately applied to the phenomenologically interesting cases. The mixing of particles and antiparticles in the meson sector (e.g. $K^0-\bar{K}^0$, $B^0-\bar{B}^0$) requires specific adjustments to the results obtained previously. Moreover, except neutrinos, all known mixed systems are subject to decay and thus the effect of particle life-time should also be taken into account.

In this short note, we analyze the phenomenological aspects of the nonperturbative field-theoretical effect in flavor mixing. Specifically, we analyze the adjustments needed for the general formulation in order to make applications for the known systems. We also study the effect of the finite particle life-time on the field-theoretical oscillation formula. Finally we estimate the magnitudes of the nonperturbative corrections in various systems and discuss the systems in which the field-theoretical effect may be most significant.

In order to illustrate the field-theoretical method, we consider the derivation of oscillation formulas for the case of mixing of neutral bosons. We begin with the mixing relations

$$\phi_A(x) = \phi_1(x)\ \cos\theta + \phi_2(x)\ \sin\theta$$
$$\phi_B(x) = -\phi_1(x)\ \sin\theta + \phi_2(x)\ \cos\theta, \tag{1}$$

where, generically, ϕ_A and ϕ_B are the fields associated with the particles with given flavor and $\phi_i(x)$ are the "free" fields with definite mass $m_{1,2}$. For the neutral particles, all fields in Eq.(1) are self-conjugate. The Fourier expansions of the free fields $\phi_{1,2}$ and their conjugate momenta $\pi_{1,2}$ are

$$\phi_i(x) = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} \frac{1}{\sqrt{2\omega_{k,i}}} \left(a_{\mathbf{k},i}\, e^{-i\omega_{k,i}t} + a^{\dagger}_{-\mathbf{k},i}\, e^{i\omega_{k,i}t} \right) e^{i\mathbf{k}\cdot\mathbf{x}} \tag{2}$$

$$\pi_i(x) = i \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} \sqrt{\frac{\omega_{k,i}}{2}} \left(a^{\dagger}_{\mathbf{k},i}\, e^{i\omega_{k,i}t} - a_{-\mathbf{k},i}\, e^{-i\omega_{k,i}t} \right) e^{i\mathbf{k}\cdot\mathbf{x}} , \tag{3}$$

where $\omega_{k,i} = \sqrt{\mathbf{k}^2 + m_i^2}$ and the nonvanishing commutators are $\left[a_{\mathbf{k},i}, a^{\dagger}_{\mathbf{p},j}\right] = \delta^3(\mathbf{k}-\mathbf{p})\delta_{ij}$ with $i,j = 1,2$.

Following Ref.[14], we recast Eq.(1) into the form

$$\phi_A(x) = G_\theta^{-1}(t)\; \phi_1(x)\; G_\theta(t) \tag{4}$$

$$\phi_B(x) = G_\theta^{-1}(t)\; \phi_2(x)\; G_\theta(t) \tag{5}$$

and similarly for $\pi_A(x)$ and $\pi_B(x)$. Here, $G_\theta(t)$ is the operator that furnishes the representation of the mixing transformation (1) in the linear space of quantum fields and can be found as

$$G_\theta(t) = exp\left[-i\,\theta \int d^3x \left(\pi_1(x)\phi_2(x) - \phi_1(x)\pi_2(x)\right)\right] . \tag{6}$$

In the finite volume, this is a unitary operator satisfying $G_\theta^{-1}(t) = G_{-\theta}(t) = G_\theta^{\dagger}(t)$ which may be written as

$$G_\theta(t) = \exp[\theta S(t)] , \tag{7}$$

with

$$S(t) = \int d^3k \left(U^*_{\mathbf{k}}(t)\, a^{\dagger}_{\mathbf{k},1} a_{\mathbf{k},2} - V^*_{\mathbf{k}}(t)\, a_{\mathbf{k},1} a_{\mathbf{k},2} + V_{\mathbf{k}}(t)\, a^{\dagger}_{\mathbf{k},1} a^{\dagger}_{\mathbf{k},2} - U_{\mathbf{k}}(t)\, a_{\mathbf{k},1} a^{\dagger}_{\mathbf{k},2} \right) . \tag{8}$$

The coefficients $U_{\mathbf{k}}(t) \equiv |U_{\mathbf{k}}|\; e^{i(\omega_{k,2}-\omega_{k,1})t}$ and $V_{\mathbf{k}}(t) \equiv |V_{\mathbf{k}}|\; e^{i(\omega_{k,1}+\omega_{k,2})t}$ are the coefficients of Bogoliubov transformation defined by

$$|U_{\mathbf{k}}| \equiv \frac{1}{2}\left(\sqrt{\frac{\omega_{k,1}}{\omega_{k,2}}} + \sqrt{\frac{\omega_{k,2}}{\omega_{k,1}}} \right) , \qquad |V_{\mathbf{k}}| \equiv \frac{1}{2}\left(\sqrt{\frac{\omega_{k,1}}{\omega_{k,2}}} - \sqrt{\frac{\omega_{k,2}}{\omega_{k,1}}} \right) . \tag{9}$$

They satisfy the unitarity relation

$$|U_{\mathbf{k}}|^2 - |V_{\mathbf{k}}|^2 = 1 , \tag{10}$$

and thus can be put in the form $|U_{\mathbf{k}}| \equiv \cosh \xi^{\mathbf{k}}_{1,2}$ and $|V_{\mathbf{k}}| \equiv \sinh \xi^{\mathbf{k}}_{1,2}$ with $\xi^{\mathbf{k}}_{1,2} = \frac{1}{2} \ln \frac{\omega_{k,1}}{\omega_{k,2}}$.

The mixing transformation also induces a $SU(2)$ coherent state structure on the quantum states [22] and the vacuum state given by

$$|0(\theta,t)\rangle_{A,B} \equiv G_\theta^{-1}(t)\; |0\rangle_{1,2} \,. \tag{11}$$

We refer to state $|0(\theta,t)\rangle_{A,B}$ as the "flavor" vacuum for the mixed fields $\phi_{A,B}$ [14].

Let us now consider the Hilbert space of the flavor fields at a given time t, say $t = 0$. It is useful to define $|0(t)\rangle_{A,B} \equiv |0(\theta,t)\rangle_{A,B}$ and $|0\rangle_{A,B} \equiv |0(\theta,t=0)\rangle_{A,B}$. In the infinite volume limit the flavor and the mass vacua are orthogonal [17]. We observe that the orthogonality disappears when $\theta = 0$ and/or $m_1 = m_2$, which is consistent with the fact that in both cases there is no mixing. For the flavor fields $\phi_{A,B}$ we then introduce the annihilation/creation operators $a_{\mathbf{k},A}(\theta,t) \equiv G_\theta^{-1}(t)\; a_{\mathbf{k},1}\; G_\theta(t)$ such that $a_{\mathbf{k},A}(\theta,t)|0(t)\rangle_{A,B} = 0$. For simplicity, we will use notation $a_{\mathbf{k},A}(t) \equiv a_{\mathbf{k},A}(\theta,t)$. Explicitly, we have

$$a_{\mathbf{k},A}(t) \;=\; \cos\theta\; a_{\mathbf{k},1} \;+\; \sin\theta\; \Big(U^*_{\mathbf{k}}(t)\; a_{\mathbf{k},2} \;+\; V_{\mathbf{k}}(t)\; a^\dagger_{\mathbf{k},2}\Big) \,, \tag{12}$$

$$a_{\mathbf{k},B}(t) \;=\; \cos\theta\; a_{\mathbf{k},2} \;-\; \sin\theta\; \Big(U_{\mathbf{k}}(t)\; a_{\mathbf{k},1} \;-\; V_{\mathbf{k}}(t)\; a^\dagger_{\mathbf{k},1}\Big) \,. \tag{13}$$

We are now in position to address the question of flavor oscillations for neutral bosons. We note that the oscillating observable should be specified properly here, because for the neutral fields all conventional charges are trivially zero($Q_{A,B} \equiv 0$). As shown in [23], however, the momentum operator for the mixing of neutral fields may be analogous to the charge operator for charged fields. In fact, if we define momentum operator for free fields by

$$\mathcal{P}_i = \int d^3x [\pi_i(x) \nabla \phi_i(x)] = \int d^3k \frac{\mathbf{k}}{2}\Big(a^\dagger_{\mathbf{k},i} a_{\mathbf{k},i} - a^\dagger_{-\mathbf{k},i} a_{-\mathbf{k},i}\Big) \tag{14}$$

and, similarly, for mixed fields,

$$\mathcal{P}_\sigma = \int d^3x [\pi_\sigma(x) \nabla \phi_\sigma(x)] = \int d^3k \frac{\mathbf{k}}{2}\Big(a^\dagger_{\mathbf{k},\sigma}(t) a_{\mathbf{k},\sigma}(t) - a^\dagger_{-\mathbf{k},\sigma}(t) a_{-\mathbf{k},\sigma}(t)\Big), \tag{15}$$

then we can show that the total momentum is conserved in time: $\mathcal{P}_A(t) + \mathcal{P}_B(t) = \mathcal{P}_1 + \mathcal{P}_2 = \mathcal{P}$. The expectation value of the momentum operator at $t \neq 0$, normalized to its initial value, is given by

$$\mathcal{P}_{\mathbf{k},\sigma}(t) \;\equiv\; \frac{{}_{A,B}\langle a_{\mathbf{k},A}|\mathcal{P}_\sigma(t)|a_{\mathbf{k},A}\rangle_{A,B}}{{}_{A,B}\langle a_{\mathbf{k},A}|\mathcal{P}_\sigma(0)|a_{\mathbf{k},A}\rangle_{A,B}} = \left|\Big[a_{\mathbf{k},\sigma}(t), a^\dagger_{\mathbf{k},A}(0)\Big]\right|^2 \;-\; \left|\Big[a^\dagger_{-\mathbf{k},\sigma}(t), a^\dagger_{\mathbf{k},A}(0)\Big]\right|^2 \tag{16}$$

$$\sigma = A, B \,. \tag{17}$$

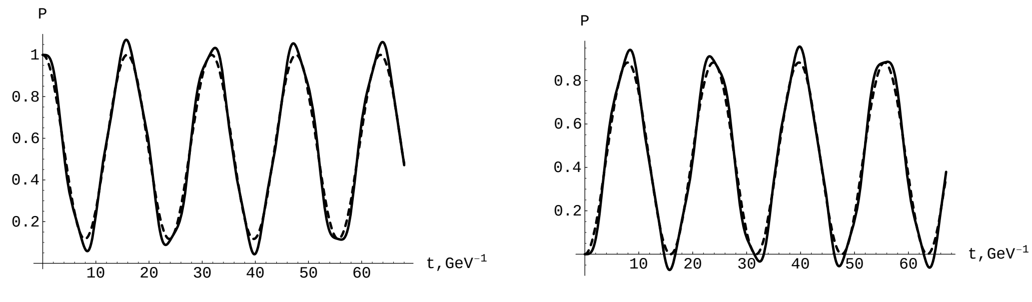

Figure 1. Relative population densities $\mathcal{P}_A$ (left) and $\mathcal{P}_B$ (right) as the function of t for $k = 0.1GeV$ in $\eta - \eta'$ system ($m_\eta = 549MeV, m'_\eta = 958MeV$ and $\theta \approx -54^\circ$ [18]). Solid line - QFT result, dashed line - QM result.

Explicitly,

$$\mathcal{P}_{\mathbf{k},A}(t) = 1 - \sin^2(2\theta)\left[|U_{\mathbf{k}}|^2 \sin^2\left(\frac{\omega_{k,2}-\omega_{k,1}}{2}t\right) - |V_{\mathbf{k}}|^2 \sin^2\left(\frac{\omega_{k,2}+\omega_{k,1}}{2}t\right)\right], \tag{18}$$

$$\mathcal{P}_{\mathbf{k},B}(t) = \sin^2(2\theta)\left[|U_{\mathbf{k}}|^2 \sin^2\left(\frac{\omega_{k,2}-\omega_{k,1}}{2}t\right) - |V_{\mathbf{k}}|^2 \sin^2\left(\frac{\omega_{k,2}+\omega_{k,1}}{2}t\right)\right]. \tag{19}$$

Eqs.(18)-(19) are the flavor oscillation formulas for the neutral mesons, such as $\eta - \eta'$, $\phi - \omega$ etc. By the definition of the momentum operator, Eqs.(18)-(19) are the relative population densities of flavor particles in the beam. As an example, $\mathcal{P}_A$ and $\mathcal{P}_B$ for the $\eta - \eta'$ system are plotted in Fig.1 as the function of time.

Still, for systems like $K_0 - \bar{K}_0$ some more care needs to be taken. Specifically, in $K^0 - \bar{K}^0$ mixing, K^0 may not be treated as neutral since $K^0 \neq \bar{K}^0$. Of course, this is not the case of mixing of two different charged particles either. Rather, the particle here is mixed with its antiparticle. To establish a connection with our previous discussion, it is important to identify the mixed degrees of freedom properly. Note that in $K^0 - \bar{K}^0$ mixing there are three distinct modes, namely the strange eigenstates $K^0 - \bar{K}^0$, the mass eigenstates $K_L - K_S$ and the CP eigenstates $K_1 - K_2$. Each pair can be written as a linear combination of the other ones, e.g.

$$\begin{array}{c} K_1 = \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0), \quad K_2 = \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0); \\ K^0 = \frac{e^{i\delta}}{\sqrt{2}}(K_L + K_S), \quad \bar{K}^0 = \frac{e^{-i\delta}}{\sqrt{2}}(K_L - K_S); \\ K_1 = \frac{1}{\sqrt{1+|\epsilon|^2}}(K_S + \epsilon K_L), \quad K_2 = \frac{1}{\sqrt{1+|\epsilon|^2}}(K_L + \epsilon K_S); \end{array} \tag{20}$$

with $e^{i\delta}$ being a complex phase and $\epsilon = i\delta$ being the imaginary CP-violation parameter. In a sense, $K^0 - \bar{K}^0$ are produced as strange eigenstates, propagate as mass eigenstates K_L, K_S and decay as CP-eigenstates K_1, K_2.

The mass eigenstates K_L and K_S are defined as the +1 and -1 CPT eigenstates, respectively, so that they can be represented in terms of self-adjoint scalar fields ϕ_1, ϕ_2 as

$$K_L = \phi_1, \quad K_S = i\phi_2. \tag{21}$$

Therefore the mixing in this system is similar to the case of neutral fields with *complex* mixing matrix. Since the complex mixing matrix in SU(2) can be always transformed into the real one by suitable redefinition of the field phases, which would not affect the expectation values, the mixing in this case is still equivalent to the mixing of neutral fields. The oscillating observables may be that of the strange charge (in the system K^0 and $\bar{K}^0$ taken as flavor A and B, respectively) with the trivial mixing angle $\theta = \pi/4$ from Eq.(20). Phenomenologically relevant, however, is the oscillation of CP-eigenvalue which determines the ratio of experimentally measured $\pi\pi$ to $\pi\pi\pi$ decay rates. CP-oscillations are given in terms of K_1 and K_2 flavors with small mixing angle $\cos\theta = 1/\sqrt{1+|\epsilon|^2}$.

The particle decay is taken in account by inserting by hand, as usually done, the factor $e^{-\gamma t}$ in the annhilation (creation) opeartors: $a_{\mathbf{k},i} \to a_{\mathbf{k},i} e^{-\frac{\gamma_i}{2}t}$. Then, the oscillation formulas can be written as

$$\begin{aligned}
\mathcal{P}_{\mathbf{k},A}(t) &= \left|\left[a_{\mathbf{k},A}(t), a^{\dagger}_{\mathbf{k},A}(0)\right]\right|^2 - \left|\left[a^{\dagger}_{-\mathbf{k},A}(t), a^{\dagger}_{\mathbf{k},A}(0)\right]\right|^2 \\
&= \left(\cos^2\theta e^{-\frac{\gamma_1}{2}t} + \sin^2\theta e^{-\frac{\gamma_2}{2}t}\right)^2 \\
&- \sin^2(2\theta) e^{-\frac{\gamma_1+\gamma_2}{2}t}\left[|U_{\mathbf{k}}|^2 \sin^2\left(\frac{\omega_{k,2}-\omega_{k,1}}{2}t\right) - |V_{\mathbf{k}}|^2 \sin^2\left(\frac{\omega_{k,2}+\omega_{k,1}}{2}t\right)\right],
\end{aligned} \tag{22}$$

$$\begin{aligned}
\mathcal{P}_{\mathbf{k},B}(t) &= \left|\left[a_{\mathbf{k},B}(t), a^{\dagger}_{\mathbf{k},A}(0)\right]\right|^2 - \left|\left[a^{\dagger}_{-\mathbf{k},B}(t), a^{\dagger}_{\mathbf{k},A}(0)\right]\right|^2 \\
&= \sin^2(2\theta)\left(\left[\frac{e^{-\frac{\gamma_1}{2}t} - e^{-\frac{\gamma_2}{2}t}}{2}\right]^2\right. \\
&+ \left. e^{-\frac{\gamma_1+\gamma_2}{2}t}\left[|U_{\mathbf{k}}|^2 \sin^2\left(\frac{\omega_{k,2}-\omega_{k,1}}{2}t\right) - |V_{\mathbf{k}}|^2 \sin^2\left(\frac{\omega_{k,2}+\omega_{k,1}}{2}t\right)\right]\right).
\end{aligned} \tag{23}$$

We note the difference between these oscillation formulas and the quantum mechanical Gell-Mann–Pais formulas. Essentially, the quantum field theoretic corrections appear as the additional high-frequency oscillation terms.

In all of field-theoretical derivations (See Eq.(18)-(23)), the field-theoretical effect (or the high-frequency oscillation term) is proportional to $|V_{\mathbf{k}}|^2$. In estimating the maximal

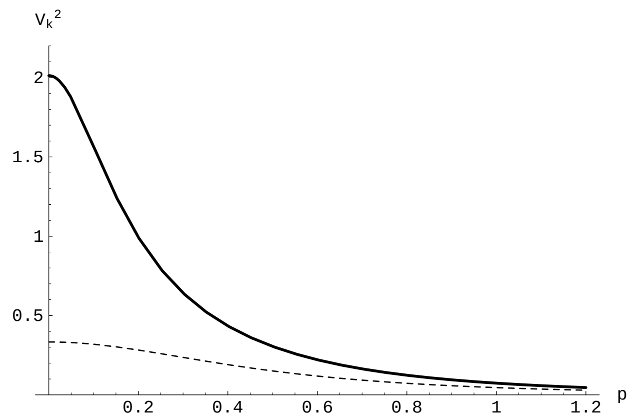

Figure 2. The bosonic condensation density $|V(p,a)|^2$ as a function of p for $a = 0.98$ (solid line) and $a = 0.8$ (dashed line).

magnitude of this term, it is useful to write $|V_{\mathbf{k}}|^2$ in terms of the dimensionless momentum $p \equiv \sqrt{\frac{2|\mathbf{k}|^2}{m_1^2+m_2^2}}$ and the dimensionless parameter $a \equiv \frac{m_2^2-m_1^2}{m_1^2+m_2^2}$ so that

$$|V(p,a)|^2 = \frac{p^2+1}{2\sqrt{(p^2+1)^2-a^2}} - \frac{1}{2}. \qquad (24)$$

As shown in Fig.2, $|V_{\mathbf{k}}|^2$ is maximal at $p = 0$ ($|V_{max}|^2 = \frac{(m_1-m_2)^2}{4m_1m_2}$) and goes to zero for large momenta (i.e. for $|\mathbf{k}|^2 \gg \frac{m_1^2+m_2^2}{2}$). The optimal observation scale for field-theoretical effect in meson mixing, therefore, is $k = 0$ and the maximal correction is of the order of $|V|^2 \sim \frac{\Delta m^2}{m^2}$. It is straightforward to find that relative field-theoretical effect in $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$, $B^0 - \bar{B}^0$ and $B_s^0 - \bar{B}_s^0$ is very small and generally does not exceed 10^{-26}. At the same time, for $\omega - \phi$ and $\eta - \eta'$ field-theoretical corrections may be as large as $5\% - 20\%$, respectively, and thus one needs to be careful about taking them into account should these systems ever be used in some sort of mixing experiments.

We can employ the similar method in the fermion sector. Since neutrinos are stable, no additional adjustments are necessary to the known results [14]. We can write the field-theoretical correction amplitude $|V_{\mathbf{k}}|^2$ as a function of the dimensionless momentum $p = \frac{|\mathbf{k}|}{\sqrt{m_1m_2}}$ and dimensionless parameter $a = \frac{m_2^2-m_1^2}{m_1m_2}$, as follows,

$$|V(p,a)|^2 = \frac{1}{2}(1 - \frac{p^2+1}{\sqrt{(p^2+1)^2+ap^2}}) \, . \qquad (25)$$

From Fig.3 we see that the effect is maximal when $p = 1$ ($|V_{max}|^2 \approx \frac{(m_1-m_2)^2}{16m_1m_2}$) and $|V|^2$

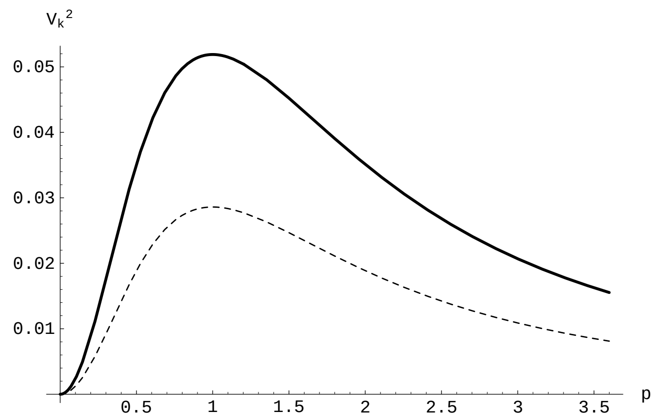

Figure 3. The fermionic condensation density $|V(p,a)|^2$ as a function of p for $a = 0.98$ (solid line) and $a = 0.5$ (dashed line).

goes to zero for large momenta (i.e. for $|\mathbf{k}|^2 \gg \frac{m_1^2+m_2^2}{2}$) as $|V|^2 \approx \frac{\Delta m^2}{4k^2}$.

Since we do not know yet the values of neutrino masses, we cannot properly specify the optimal scale for observation of field-theoretical effect in this sector. However, certainly this scale cannot be much larger than a fraction of eV. So far the experimentally observed neutrinos are always extremely relativistic and, therefore, the value of $|V|^2$ may be estimated as $|V|^2 \sim \frac{\Delta m^2}{k^2} \sim 10^{-18}$. Only for extremely low energies (like those in neutrino cosmological background) the field-theoretical corrections might be large and account for few percent. In this connection, we observe that the non-perturbative field theory effects, in spite of the small corrections they induce in the oscillation amplitudes, nevertheless they may contribute in a specific and crucial way in other physical contexts or phenomena. An example of this is provided by the recent result [24] which shows that the mixing of neutrinos may specifically contribute to the value of the cosmological constant exactly because of the non-perturbative effects expressed by the non-zero value of $|V_{\mathbf{k}}|^2$.

To summarize, in this note we considered phenomenological aspects of the quantum field theoretical formalism for spin-zero boson-field mixing. A crucial point in our analysis is the disclosure of the fact that the space for the mixed field states is unitarily inequivalent to the state space where the unmixed field operators are defined. This is a common feature with the QFT structure of mixing, which has recently been established. The vacuum for the mixed fields turns out to be a generalized $SU(2)$ coherent state.

We have estimated the magnitude of the field-theoretical effect in known mixed systems.

We found that for most of known mixed systems both in meson and neutrino sectors this effect is negligible. Only in strongly mixed systems, such as $\omega-\phi$ or $\eta-\eta'$, or for very low-energy neutrino effects the corrections may be as large as $5\% - 20\%$ and thus additional attention may be needed if these systems can be used in oscillation experiments. The non-perturbative vacuum effect is the most prominent when the particles are produced at low momentum.

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