Charging and Discharging of a Capacitor

Objective
Become familiar with the charging and discharging of a capacitor in an electric circuit.

Theory
Capacitor is an electric device for storing electric charge. Capacitor is usually made of two or more conducting surfaces placed close to each other inside a dielectric filling, used to decrease the electric field inside the capacitor. When the surfaces are connected to a power source such as an external battery, the positive charge develops on one of the surface of the capacitor and equal negative charge develops on the other surface, in order to compensate for the applied voltage. When the battery is disconnected, the capacitor can retain this charge because of the electric attraction between close positive and negative charges on the surfaces inside it.

Capacitors are characterized by their capacitance, \( C \), defined as the ratio of the charge stored in the capacitor, \( Q \), and the potential difference between its plates, \( V, C = Q/V \). Capacitance measures how easy it is to store a charge in a capacitor – higher capacitance means more charge can be stored using the same voltage \( V \).

When a discharged capacitor is first connected to an external battery, or when a charged capacitor is first put into a circuit, it undergoes a charging or discharging process as the charges are moved in or out of the capacitor. This process takes time, and can be described by the following equation obtained from the Kirchhoff’s loop rule,

\[
V_0 + IR - \frac{Q}{C} = 0
\]

![Figure 1](image.png)

Figure 1 In capacitor circuit Kirchhoff’s loop rule states that the sum of battery’s voltage, voltage drop on the resistor, and voltage drop on the capacitor should all add to zero, \( V_0 + IR - \frac{Q}{C} = 0 \).

The importance of this equation is to relate the current in the circuit \( I \) at certain time with the charge stored in the capacitor at that time \( Q \). Since the current \( I \) defines how fast the capacitor is charging, in fact since \( I = \frac{dQ}{dt} \), \( I \) is exactly the time derivative of \( Q \), \( I(t) = Q'(t) \).

Considering this, it can be shown that the solution to the above equation have to behave with time according to the following two sets of equations:

\[
Q(t) = V_0C \cdot (1 - e^{-t/RC})
\]

\[
I(t) = \frac{V_0}{R} e^{-t/RC} \quad \text{(charging)}
\]
\[ Q(t) = V_0 e^{-t/RC} \]
\[ I(t) = -\frac{V_0}{R} e^{-t/RC} \] (discharging)

You can check that these equations do satisfy \( V_0 + I(t)R - Q(t)/C = 0, I(t) = Q'(t) \) at all times. Note also that the potential on the capacitor also changes with the time proportionally to \( Q \), that is \( V_C(t) = Q(t)/C \).

As can be seen from these solutions, the capacitor can charge or discharge only with certain speed defined by the time-constant \( \tau = RC \). This constant is called the capacitive time constant.

Figure 2 As a solution to Kirchhoff’s rule, the charge and the voltage on the capacitor during charging should grow exponentially over time \( \tau \approx RC \). Similarly, during discharging the charge and the voltage should drop exponentially over the same time.

Other than the capacitor charging and discharging time, this same time constant has also a more general meaning in relation to the time needed for charges in any conductor, such as a metal sphere placed into an external electric field, to re-arrange themselves on the surface of the conductor to cancel out the electric field inside it. This process can be viewed as the charging of a capacitor above, and takes the same amount of time \( \tau = RC \). In this context, \( \tau \) is also sometimes called the relaxation time.

**Equipment**
- Electric circuit experiment set.
- Electric cables.
- Multimeter.
- Two resistor elements \( R = \) __________.
- Two capacitor elements, \( C = \) __________ and \( C = \) __________.
Procedures
1. Make sure that the power is turned off. Implement the circuit in diagram A using $C=______$ capacitor, and provided multimeter in “DC Voltage” mode. **NOTE:** Once complete, verify your circuit with the instructor and obtain permission to proceed.
   a. Turn the power on and use the multimeter to take the measurements of the voltage on the capacitor every 5 seconds for approximately two minutes as the capacitor is charging.
   b. Wait for the capacitor to charge completely. Then, turn the power off and use the multimeter to take the measurements of the voltage on the capacitor every 5 seconds for approximately two minutes as the capacitor is discharging.
2. Implement the circuit in diagram B using the capacitors and the resistors connected in series, and repeat the measurements in 1a-1b.

**ALYSIS (TO BE PERFORMED IN THE REPORT)**
3. Make a graph for the voltage you measured in 1.a and 1.b as a function of time. Is it exponential?
4. For your measurements in 1.a, make a graph of $f = \ln \left(1 - \frac{V(t)}{V_{max}}\right)$ using as $V_{max}$ the maximal voltage that you observed. Fit a straight line to this graph and find its slope $k$. Use $k$ to calculate the capacitive time constant $\tau_{exp} = 1/k$ and compare it with the theoretical value $\tau_{theory} = RC$.
5. For your measurements in 1.b, make a graph of $f = \ln V(t)$. Fit a straight line to this graph and find its slope $k$. Use $k$ to calculate the capacitive time constant $\tau_{exp} = 1/k$ and compare it with the theoretical value $\tau_{theory} = RC$.
6. Calculate the theoretical value for capacitive time constant $\tau_{theory}$ in circuit B in 2).
7. Repeat steps 4-5 for your measurements in 2), and compare the experimental value $\tau_{exp}$ with that you calculated in 6) theoretically.

**ELECTRIC CIRCUIT DIAGRAMS**

*Circuit A;*

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+------------------+
|                  |
|   R1             |
|                  |
|  C1              |

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*Circuit B;*

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+------------------+
|                  |
|   R1             |
|                  |
|  C1              |
|                  |
|  R2              |
|  C2              |

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